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Consecutive Magic Labeling of N Copies of Generalized Petersen Graphs

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ABSTRACT

Let $G = (V, E)$ be a simple graph with v vertices and e edges. A bijection $f : E \to \{1, 2,\ldots e\}$ such that weight of vertices of G constitute a set of consecutive integers is called a consecutive magic labeling and G is called consecutive magic where weight of a vertex is the sum of labels of the edges incident to it. In this paper we study the consecutive magicness of N copies of generalized Petersen graphs. A generalized Petersen graph $P(n, m)$, $n \geq 3$, $1 \leq m < \frac{n}{2}$ is a 3-regular graph with $2n$ vertices $u_1, u_2,\ldots u_n, v_1, v_2,\ldots v_n$ and edges (u_i, v_i), $(u_i, u_i + 1)$, $(v_i, v_i + m)$ for all $i \in \{1, 2,\ldots n\}$, where the subscripts are taken modulo n. $P(5, 2)$ is the standard Petersen graph.

Keywords: Consecutive Magic Labeling, Copywise Consecutive Magic, Generalized Petersen Graphs

INTRODUCTION

A very popular concept of Graph theory is the concept of labeling of graphs which was introduced in the late 1960's. Graph labeling is an assignment of integers to the vertices or edges or both under certain conditions. A detailed survey of graph labelings is given in [2]. In this paper, we study the consecutive magic labeling of N copies of Generalized Petersen graphs where N is a finite positive integer. Various consecutive magic labelings are discussed in [1].

Definition* 1*: Let $G = (V, E)$ be a simple graph with v vertices and e edges. A bijection $f : E \to \{1, 2,\ldots e\}$ such that weight of vertices of G constitute a set of consecutive integers is called a consecutive magic labeling and G is called consecutive magic where weight of a vertex is the sum of labels of the edges incident to it.

***Definition* 2**: A generalized Petersen graph $P(n, m)$, $n \geq 3$, $1 \leq m < \frac{n}{2}$ is a 3-regular graph with $2n$ vertices $u_1, u_2,\ldots u_n, v_1, v_2,\ldots v_n$ and edges (u_i, v_i), (u_i, u_{i+1}), (v_i, v_{i+m}) for all $i \in \{1, 2,\ldots n\}$, where the subscripts are taken modulo n.

***Definition* 3:** Let $G = (V, E)$ be a simple graph with v vertices and e edges. Consider NG, the N copies of G where N is a finite positive integer. A bijection g: $E(NG) \to \{1, 2,\ldots Ne\}$ is called *copywise consecutive magic labeling* if each t^{th} copy of NG is consecutive magic separately with its own set of weight of vertices W_t constituting a set of consecutive integers and NG is called *copywise consecutive magic*.

Theorem 1: *If n is odd,* $n \geq 3$, $m \leq \left\lfloor \frac{n}{2} \right\rfloor$ *and N is a finite positive integer then N P(n, m) is not copywise consecutive magic .*

Proof: In [3], we see that if n is odd, $n \geq 3$, and $m \leq \left\lfloor \frac{n}{2} \right\rfloor$ then $P(n,m)$ is not consecutive magic. Hence clearly $N\,P(n, m)$ is not copywise consecutive magic for a finite positive integer N.

Consecutive-magic labeling of a generalized Petersen graph *P(n,m).*

Theorem 2: *For* $n \geq 4$, *n even and* $m \leq \frac{n}{2} - 1$, *the generalized Petersen graph P(n, m) has a consecutive magic labeling.*

Proof: Define $f : E \to \{1, 2,\ldots e = 3n\}$ as

$$f(u_i u_{i+1}) = \begin{cases} \dfrac{5n+i+1}{2} & \text{if } 1 \leq i \leq n-1 \text{ is odd} \\ \dfrac{n-i+2}{2} & \text{if } 2 \leq i \leq n \text{ is even} \end{cases}$$

$$f(u_i v_i) = \begin{cases} \dfrac{3n-i+1}{2} & \text{if } 1 \leq i \leq n-1 \text{ is odd} \\ n+\dfrac{i}{2}-2 & \text{if } 2 \leq i \leq n \text{ is even} \\ \dfrac{n+i-4}{2} & \text{if } 6 \leq i \leq n \text{ is even} \end{cases}$$

$$f(v_i v_{i+m}) = \begin{cases} \frac{3n+i+1}{2} & if\ 1 \le i \le n-1 \text{ is odd} \\ 2n+3-\frac{i}{2} & if\ 2 \le i \le n \text{ is even} \\ \frac{5n-i}{2}+3 & if\ 6 \le i \le n \text{ is even} \end{cases}$$

Then we can easily see that the set of weights of vertices are given by

$$w(u_i) = \left\{\frac{7n}{2}+2, \frac{7n}{2}+3, \ldots 4n-1, 4n, 4n+1, \ldots \frac{9n}{2}, \frac{9n}{2}+1\right\}$$

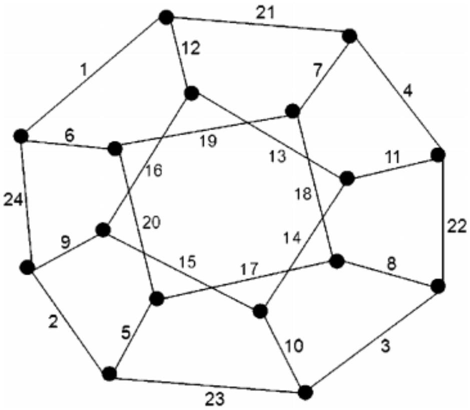
$$w(v_i) = \left\{\frac{9n}{2}+2, \frac{9n}{2}+3, \ldots, \frac{11n}{2}+1\right\}.$$

Thus the weights of vertices of $P(n,m)$ constitute the set of consecutive integers

$$\left\{\frac{7n}{2}+2, \frac{7n}{2}+3, \ldots \frac{9n}{2}, \frac{9n}{2}+1 \ldots \frac{11n}{2}+1\right\}.$$

Example

Consecutive Magic Labeling of *P* (8, 2)



***W* = { 30, 31, ..., 45 }**

Copywise Consecutive Magic Labeling of N*P*(*n*,*m*)

Theorem 3: *If $n \ge 4$, n even, $m \le \frac{n}{2}-1$ and N is a finite positive integer then N P(n, m) is copywise consecutive magic with the set of weight of vertices W_t of the t th copy given by*

$$W_t = \left\{\frac{[6(N+t)-5]n}{2}+2, \frac{[6(N+t)-5]n}{2}+3, \ldots, \frac{[6(N+t)-1]n}{2}+1\right\},$$

where t = 1,2,…N.

Proof: Label the first graph of $NP(n, m)$ as given in theorem 2. Label each of the t^{th} graph (copy) by adding each label with $(t-1)3n$. Now interchange the corresponding edge labels of the $u_i v_i$ edges in the first graph with the ones in the N^{th} graph. Do similarly between the second graph and the $(N-1)^{th}$ graph, the third graph and the $(N-2)^{nd}$ graph and so on, except in the case of N being odd where the labels of $(N+1)/2^{nd}$ graph remains unaltered.

In this manner, we construct the labeling g of $NP(n,m)$ as follows: Consider the labeling f given in theorem 2.Then the labeling of the t^{th} graph (copy) of $NP(n,m)$ is given by

$$g^t(u_i v_i) = f(u_i v_i) + (N-t)3n$$

$$g^t(u_i u_{i+1}) = f(u_i u_{i+1}) + (t-1)3n$$

$$g^t(v_i v_{i+m}) = f(v_i v_{i+m}) + (t-1)3n, \text{ where } t = 1,2,\ldots N$$

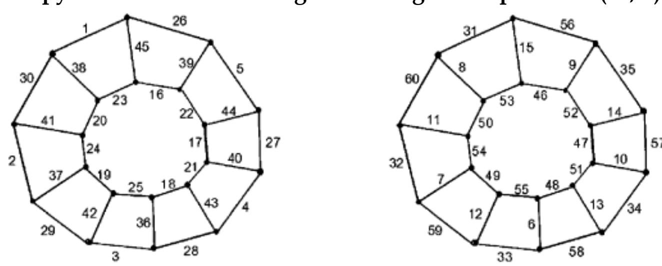
Then it is easily seen that the set of weight of vertices W_t of the t^{th} copy is given by

$$W_t = \left\{\frac{[6(N+t)-5]n}{2}+2, \frac{[6(N+t)-5]n}{2}+3, \ldots, \frac{[6(N+t)-1]n}{2}+1\right\}$$

where $t = 1, 2, \ldots N$.

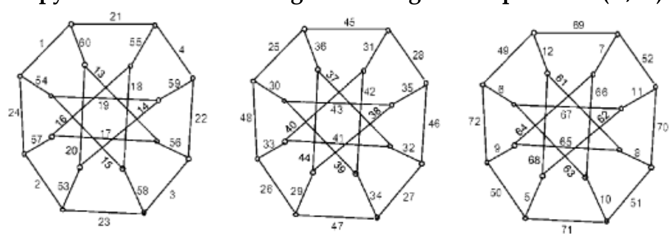
Examples

Copywise Consecutive Magic Labeling of 2 copies of *P* (10, 1)



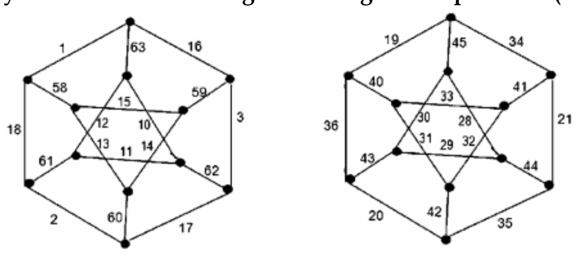
W_1 = { 67, 68, … , 86 } W_2 = { 97, 98, … , 116 }

Copywise Consecutive Magic Labeling of 3 Copies of *P* (8, 3)

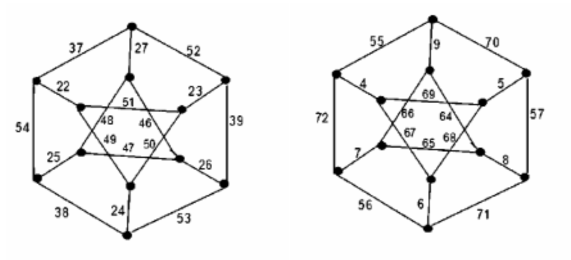


W_1 = { 78, 79, …, 93 } W_2 = { 102, 103, … , 117 } W_3 = { 126, 127, … , 141 }

Copywise Consecutive Magic Labeling of 4 Copies of *P* (6, 2)

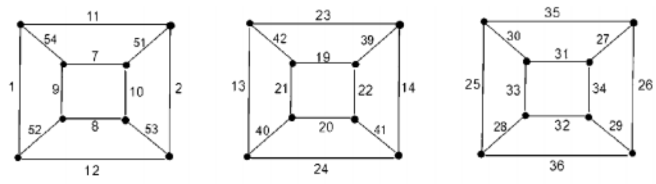


W_1 = { 77, 78, …, 88 } W_2 = { 95, 96, …, 106 }

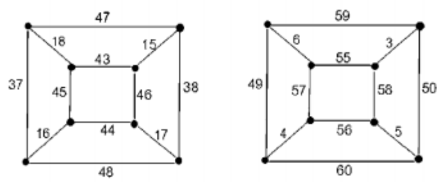


W_3 = { 113, 114, …, 124 } W_4 = { 131, 132, …, 142 }

Copywise Consecutive Magic Labeling of 5 Copies of *P* (4, 1)



W_1 = { 64, 65, …, 71 } W_2 = { 76, 77, …, 83 } W_3 = { 88, 89, …, 95 }



W_4 = { 100, 101, …, 107 } W_5 = { 112, 113, …, 119 }

CONCLUSION

In this paper we have constructed copywise consecutive magic labeling for N copies of Generalized Petersen graphs which are isomorphic to each other. Still construction of such labeling remains open in the case of non isomorphic disconnected graphs.

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