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A Production Lot-size Inventory Model for Weibull Deteriorating Item with Quadratic Demand, Quadratic Production and Shortages

A. Kalam[1], D. Samal[2], S. K. Sahu[3] & M. Mishra[4]

[1]Department of Mathematics, Gandhi Institute For Technology, Bhubaneswar, Orissa, India, Pin-752054
[2]Department of Mathematics, College of Engineering Bhubaneswar-24, Orissa.
[3]National Institute of Financial Management, Faridabad, Delhi, India. Pin-121001
[4]Principal, Govt Science College Malkanagiri, Orissa.
[1]abdul_003m@rediffmail.com

ABSTRACT

In this paper, we consider the production inventory problem in which the deterioration is weibull distribution; production and demand are quadratic function of time. Shortages of cycle are allowed in the inventory system. The solution of the model is discussed for finite time horizon. The numerical example is taken up to illustrate the solution procedure and sensitivity analysis is also carried out.

Keywords: Inventory; Deterioration; Quadratic Demand; Quadratic Production; Shortages.

1. INTRODUCTION

In general, deterioration is defined as the damage, spoilage, dryness, vaporization, etc., that result in decrease of usefulness of the original one. Goods deteriorate and their value reduces with time. Electronic products may become absolute as technology changes. Fashion tends to depreciate the value of clothing over time .Batteries dies out as they age. The effect of time is even more critical for perishable goods such as foods stuffs and cigarettes. The effect of deterioration and time is that the classical inventory model has to be readjusted. The decrease or loss of utility due to decay is usually a function of the on-hand inventory. It is reasonable to note that a product may be understood to have life time, which ends when utility reaches zero.

Haiping and Wang [7] developed an economic policy model for deteriorating items with time proportional demand. Donaldson [8] derived an analytical solution to the problems of obtaining the optimal number of replenishments and the optimal replenishment times of an EOQ model with a linearly time dependent demand pattern over a finite time horizon. Zangwill [9] developed a discrete-in-time dynamic programming algorithm to solve an inventory model by allowing the inventory levels to be negative where the demand pattern is time dependent. Following the approach of Donaldson [8], Murdeshwar [6] has tried to derive an exact solution for a finite horizon inventory model to obtain the optimal number of replenishments, optimal replenishments times and the optimal times at which the inventory level falls to Zero, assuming the demand rate to be linearly time dependent and shortages. Hamid [3], Kun-Shan Wu et.al [5] presented a heuristic model for determining the ordering schedule when inventory items are subjects to deterioration and demand changes linearly over time and obtained an optimal replenishment cycle length Goswami and Chaudhuri [2] presented an EOQ model deteriorating items with shortage and linear trend in demand. Brad Shaw and Erol [1] published a paper in which they derived unbounded control policies for a class of linear time invariant production- inventory systems.

In this paper, inventory production system where items follow Weibull deterioration, Quadratic demand, Quadratic production and shortages. The objective is to develop optimal policy that minimizes the cost associated with the inventory and production rate for finite time horizon. A numerical example is given conduced to study the affect of cost parameter on the objective function.

2. ASSUMPTIONS AND NOTATIONS

1. The lead-time is Zero and shortages are allowed.
2. Planning horizon is finite.
3. The demand rate is assumed to be $R(t) = a + bt + ct^2$, where being constants.
4. The production rate say $k = \gamma R(t)$, where $\gamma > 1$.
5. The fraction of the on-hand inventory deteriorates per unit time, where

   $Z(t) = \alpha\beta t^{\beta-1}$, $0 < \alpha < 1$, $t > 1$ and $\beta \geq 1$.
6. Unit holding cost c_1 per unit time, unit deterioration cost c_2 per unit time, unit shortage

cost c_3 per unit time and unit opportunity cost c_4 per unit time are known and constant.

7. c is the total average cost for the production cycle and S is the stick level reached in the cycle .

3. MATHEMATICAL FORMULATION

Let N be the inventory level at any time $t(0 \le t \le t_3)$.. The differential equations governing the system in the interval $(0\ t_2)$ are

$$\frac{dN}{dt} + Z(t)N = k - R(t),\ 0 \le t \le t_1 \quad (1)$$

$$\frac{dN}{dt} + Z(t)N = R(t),\ t_1 \le t \le t_2 \quad (2)$$

The stock level initially is zero. Production begins just after $t = 0$, continues up to $t = t_1$ and stops as soon as the stock level becomes S. then the inventory level decrease due to demand and deterioration both till it becomes zero at $t = t_2$. The cycle then repeats itself. The inventory of deterioration is very low initially but it increase with time. However, it remains bounded for. $t \ge 1$.

Using the values of $R(t)$ and $Z(t)$, the two equations (1) and (2) take the form.

$$\frac{dN}{dt} + \alpha\beta t^{\beta-1} N = (\gamma - 1)(a + bt + ct^2), 0 \le t \le t_1 \quad (3)$$

$$\frac{dN}{dt} + \alpha\beta t^{\beta-1} N = -(a + bt + ct^2),\ t_1 \le t \le t_2 \quad (4)$$

The solution of equation (3) with initial conditions is

$$N = (\gamma - 1)\left(at + \frac{bt^2}{2} + \frac{ct^3}{3} - \frac{a\alpha t^{\beta+1}}{\beta+1} - \frac{b\alpha\beta t^{\beta+2}}{2(\beta+2)} - \frac{c\alpha\beta t^{\beta+3}}{3(\beta+3)} \right) \quad (5)$$

Neglecting the power of α greater than one. Similarly the solution of equation (4) also is neglecting the powers of α greater than one.

$$N = c^1(1 - at^b)\left(at + \frac{bt^2}{2} + \frac{ct^3}{3} - \frac{a\alpha t^{\beta+1}}{\beta+1} - \frac{b\alpha\beta t^{\beta+2}}{2(\beta+2)} - \frac{c\alpha\beta t^{\beta+3}}{3(\beta+3)} \right) \quad (6)$$

For $t = t_1$, $N = S$

$$S = c^1(1 - \alpha t_1^{\beta})\left(at_1 + \frac{bt_1^2}{2} + \frac{ct_1^3}{3} - \frac{a\alpha t_1^{\beta+1}}{\beta+1} - \frac{b\alpha\beta t_1^{\beta+2}}{2(\beta+2)} - \frac{c\alpha\beta t_1^{\beta+3}}{3(\beta+3)} \right) \quad (7)$$

From equations (6) and (7), we get the relation

$$N = S(1 + \alpha t_1^{\beta} - \alpha t^{\beta}) + a$$

$$\left(t_1 - t + \frac{\alpha\beta t^{\beta+1}}{\beta+1} + \frac{\alpha\beta t_1^{\beta+1}}{\beta+1} - \alpha t_1 t^{\beta} \right) + b\left(\frac{1}{2}(t_1^2 - t^2) \right) +$$

$$\frac{\alpha\beta t^{\beta+2}}{2(\beta+2)} + \frac{\alpha\beta t_1^{\beta+2}}{\beta+2} - \frac{\alpha t_1^2 t^{\beta}}{2}$$

$$+ c\left(\frac{1}{3}(t_1^3 - t^3) + \frac{\alpha\beta t^{\beta+3}}{2(\beta+3)} + \frac{\alpha\beta t_1^{\beta+3}}{\beta+3} - \frac{\alpha t_1^3 t^{\beta}}{3} \right) \quad (8)$$

Using the condition $N = 0$ for $t = t_2$ in equation (8), we get

$$S = a\left(t_2 - t_1 + \frac{\alpha\beta t_2^{\beta+1}}{\beta+1} + \frac{\alpha\beta t_1^{\beta+1}}{\beta+1} - \alpha t_1^{\beta} t_2 \right)$$

$$+ b\left(\frac{t_2^2 - t_1^2}{2} + \frac{\alpha t_2^{\beta+2}}{\beta+2} + \frac{\alpha\beta t_1^{\beta+2}}{\beta+2} - \frac{\alpha t_1^{\beta} t_2^{\beta}}{2} \right)$$

$$+ c\left(\frac{t_2^3 - t_1^3}{3} + \frac{\alpha t_2^{\beta+3}}{\beta+3} + \frac{\alpha\beta t_1^{\beta+3}}{\beta+3} - \frac{\alpha t_1^{\beta} t_2^3}{3} \right) \quad (9)$$

During the shortage interval $[t_2, t_3]$, the demand at time t is partially backlogged at fraction $B(t_3 - t)$. thus, the inventory level at time t, is governed by the following differential equation

$$\frac{dN}{dt} = -\alpha\beta(t_3 - t) = -\frac{\alpha}{1 + \delta(t_3 - t)},\ t_2 \le t \le t_3 \quad (10)$$

With the boundary condition $N(t_2) = 0$ and neglect the power of δ greater than one. The solution of Eq.(10) is

$$N = \alpha(t_2 - t) + \frac{\alpha\delta}{2}(t_2^2 - t^2 - 2t_2t_3 + 2tt_3), t_2 \le t \le t_3 \quad (11)$$

The ordering cost per cycle = A

The inventory holding cost per cycle is given by

$$HC = c_1\left[\int_0^{t_1} N(t)dt + \int_{t_1}^{t_2} N(t)dt \right]$$

Now, substituting the values of S from (9) and simplify we get,

$$HC = c_1\left[\frac{a}{2}\left(\gamma t_1^2 - \frac{2\alpha\beta\gamma t_1^{\beta+2}}{(\beta+1)(\beta+2)} + t_2^2 - 2t_1t_2 \right.\right.$$

$$\left. + \frac{2\alpha\beta t_2^{\beta+1}}{(\beta+1)(\beta+2)} - \frac{2\alpha t_1 t_2^{\beta+1}}{\beta+1} + \frac{2\alpha t_2 t_1^{\beta+1}}{\beta+1} \right)$$

$$+ \frac{b}{6}\left(\gamma t_1^3 + 2t_2^3 - 3t_1t_2^2 + \frac{6\alpha\beta t_2^{\beta+3}}{(\beta+1)(\beta+3)} \right.$$

$$\left. - \frac{6\alpha t_1 t_2^{\beta+2}}{\beta+2} + \frac{3\alpha t_1^{\beta+1} t_2^2}{\beta+1} - \frac{3\alpha\beta\gamma t_1^{\beta+3}}{(\beta+2)(\beta+3)} \right)$$

$$\frac{c}{12}\left(\gamma t_1^4 + 3t_2^4 - 4t_1t_2^3 + \frac{12\alpha\beta t_2^{\beta+4}}{(\beta+1)(\beta+4)} \right.$$

$$\left.\left. - \frac{12\alpha t_1 t_2^{\beta+3}}{\beta+3} + \frac{4\alpha t_1^{\beta+1} t_2^3}{\beta+1} - \frac{3\alpha\beta\gamma t_1^{\beta+4}}{(\beta+3)(\beta+2)} \right) \right] \quad (12)$$

The deterioration cost per cycle is given by

$$DC = c_2\left[\gamma\int_0^{t_1}\left(a+bt+ct^2\right)dt - \int_0^{t_2}\left(a+bt+ct^2\right)dt\right]$$

$$= c_2\left[a\left(\gamma t_1 - t_2\right)+\frac{b}{2}\left(\gamma t_1^2 - t_2^2\right)+\frac{c}{3}\left(\gamma t_1^3 - t_2^3\right)\right] \qquad (13)$$

In equation (11) the shortage cost per cycle due to backlog is given by

$$SC = c_3\int_{t_2}^{t_3}[-N(t)]dt$$

$$= c_3\left[\frac{\alpha}{2}(t_2-t_3)^2 - \beta\delta\left\{\frac{1}{3}\left(t_3^3-t_2^3\right)+t_2^2t_3 - t_2t_3^2\right\}\right] \qquad (14)$$

The opportunity cost per cycle due to lost sales is given by

$$OC = c_4\int_{t_2}^{t_3}\alpha\left[1-B(t_3-t)\right]dt$$

$$= c_4\,\frac{\alpha\delta}{2}(t_3-t_2)^3 \qquad (15)$$

From equation. (12)-(15) the total average cost of the inventory is given by

$$C = \frac{\left(HC+DC+SC+OC\right)}{t_3}$$

$$C(t_2,t_3) = C_1\left[\frac{a}{2}\left(\gamma t_1^2 - \frac{2\alpha\beta\gamma t_1^{\beta+2}}{(\beta+1)(\beta+2)} + t_2^2 - 2t_1t_2 + \frac{2\alpha\beta t_2^{\beta+1}}{(\beta+1)(\beta+2)} - \frac{2\alpha t_1 t_2^{\beta+1}}{\beta+1} + \frac{2\alpha t_2 t_1^{\beta+1}}{\beta+1}\right) + \frac{b}{6}\left(\gamma t_1^3 + 2t_2^3 - 3t_1t_2^2 + \frac{6\alpha\beta t_2^{\beta+3}}{(\beta+1)(\beta+3)} - \frac{6\alpha t_1 t_2^{\beta+2}}{\beta+2} + \frac{3\alpha t_1^{\beta+1}t_2^2}{\beta+1} - \frac{3\alpha\beta\gamma t_1^{\beta+3}}{(\beta+2)(\beta+3)}\right)\right.$$
$$\left. + \frac{c}{12}\left(\gamma t_1^4 + 3t_2^4 - 4t_1t_2^3 + \frac{12\alpha\beta t_2^{\beta+4}}{(\beta+1)(\beta+4)} - \frac{12\alpha t_1 t_2^{\beta+3}}{\beta+3} + \frac{4\alpha t_1^{\beta+1}t_2^3}{\beta+1} - \frac{3\alpha\beta\gamma t_1^{\beta+4}}{(\beta+3)(\beta+4)}\right)\right]$$

$$c_2\left[a\left(\gamma t_1 - t_2\right)+\frac{b}{2}\left(\gamma t_1^2 - t_2^2\right)+\frac{c}{3}\left(\gamma t_1^3 - t_2^3\right)\right] + c_3\left[\frac{\alpha}{2}(t_2-t_3)^2\right.$$
$$\left. -\alpha\delta\left\{\frac{1}{3}\left(t_3^3-t_2^3\right)+t_2^2t_3 - t_2t_3^2\right\}\right] + c_4\,\frac{\alpha\delta}{2}(t_3-t_2)^3 \qquad (16)$$

The necessary conditions for the total average cost per unit time in equation (16) to be minimum are $\frac{\partial c}{\partial t_2}=0, \frac{\partial c}{\partial t_3}=0$

$$\Rightarrow c_1\left[a\left(t_2 - t_1 + \frac{\alpha\beta}{\beta+1}t_2^{\beta+1} - \alpha t_1 t_2^{\beta} + \frac{\alpha}{\beta+1}t_1^{\beta+1}\right) + b\left(t_2^2 - t_1t_2 + \frac{\alpha\beta}{\beta+1}t_2^{\beta+2} - \alpha t_1 t_2^{\beta+1} + \frac{\alpha}{\beta+1}t_1^{\beta+1}t_2\right) + c\left(t_2^3 - t_1t_2^2 + \frac{\alpha\beta}{\beta+1}t_2^{\beta+3} - \alpha t_1 t_2^{\beta+1} + \frac{\alpha}{\beta+1}t_2^2 t_1^{\beta+1}\right)\right]$$

$$-c_2\left(a - bt_2 - ct_2^2\right) + c_3\left(\alpha\left(t_2-t_3\right)+\alpha\delta t_2^2 + 2t_2t_3 - t_3^2\right) - \frac{3}{2}c_4\,\alpha\delta\left(t_3-t_2\right)^2 = 0 \qquad (17)$$

$$c_3\left(\alpha\left(t_2-t_3\right)+\alpha\delta\left(t_3^2+t_2^2-2t_2t_3\right)\right) - \frac{3}{2}c_4\alpha\delta\left(t_3-t_2\right)^2 + C = 0 \qquad (18)$$

The optimum value of t_2 and t_3 can be obtained by solving the above non-linear expressions along MATHEMATICA 5.2.provided that satisfy the sufficient conditions

$$\frac{\partial^2 C}{\partial t_2^{\,2}} > 0, \frac{\partial^2 C}{\partial t_3^{\,2}} > 0 \text{ and}$$

$$\frac{\partial^2 C}{\partial t_2^{\,2}}\frac{\partial^2 C}{\partial t_3^{\,2}} - \left[\frac{\partial^2 C}{\partial t_2 \partial t_3}\right]^2 > 0$$

4. NUMERICAL EXAMPLE

Let $\alpha = 0.0001$, $\beta = 1.5$, $\gamma = 2$, $c_1 = 4$, $c_2 = 15$, $c_3 = 60$, $c_4 = 40$, $a = 250$, $b = 10$, $c = 12$, in suitable units. . By applying the subroutine find root in MATHEMATICA 5.2.we obtain the optimal solution for values of t_2 and t_3 is $t_2^* = 6.462$ $t_3^* = 7.671$, which gives minimum average cost .

cost $C^* = 8456.28$.

Table

parameters	*% change*	t_2*	t_2*	C*
α	50	6.287	7.618	8606.89
	25	6.238	7.678	8625.05
	– 25	6.145	7.656	8631.14
	– 50	6.119	7.642	8639.20
β	50	6.485	7.893	8495.44
	25	6.484	7.891	8497.07
	– 25	6.482	7.889	8506.79
	– 50	6.481	7.887	8540.87
γ	50	4.035	5.481	3511.37
	25	4.553	7.076	4565.84
	-25	4.885	8.765	5876.16

Contd...

Contd...

	– 50	5.119	8.879	8668.34
a	50	8.807	7.105	6594.07
	25	6.253	7.321	6897.35
	– 25	6.282	7.689	7224.10
	– 50	7.032	8.179	7553.41
b	50	8.413	9.491	7189.36
	25	8.528	9.689	7236.78
	– 25	8.692	9.843	7246.38
	– 50	9.131	9.992	7282.29
c	50	8.630	9.018	7068.57
	25	8.282	8.698	6906.78
	– 25	7.103	7.328	6826.14
	– 50	6.454	6.625	6741.38
c_1	50	7.282	70869	6886.61
	25	8.025	9.934	7106.79
	– 25	9.760	10.352	7900.92
	– 50	10.035	12.050	8960.35
c_2	50	8.893	9.689	7106.78
	25	7.638	7.769	7491.38
	– 25	7.277	7.587	7835.85
	– 50	7.023	7.423	8397.43
c_3	50	8.963	9.683	7306.92
	25	8.482	9.321	7535.34
	– 25	7.834	8.843	7892.85
	– 50	7.343	8.527	8132.91
c_4	50	9.612	9.883	7589.41
	25	9.531	9.681	7823.58
	– 25	9.253	9.354	8152.19
	– 50	8.861	9.157	8471.43

5. SENSITIVITY ANALYSIS

Sensitivity analysis is performed by changing (increasing or decreasing) the parameters by 25% and 50% and taking one parameter at a time keeping the remaining parameters at their original values.

a. The optimal length of inventory depends upon quadratic demand. The average cost the system per unit time C^* increase with decrease in the value of parameters α, β, γ, a, b, c_1, c_2, c_3, and c_4, C^* decrease with decrease in the value of parameter c .

b. The optimal length of inventory increase with decrease in the value of parameters γ, a, b, and c_1.

The optimal length of inventory decrease with decrease in the value of Parameters α, β, c, c_2, c_3 and c_4.

6. CONCLUSION

In this paper a deterministic inventory model has been proposed for quadratic demand rate, quadratic production and weibull deterioration rate with shortages are consider. Sensitivity analysis of the results and the total average cost for a production cyclic C^* decrease in the value of parameters α, β, γ, a, b, c_1, c_2, c_3, c_4, C^* decrease in the value of parameter c. The future extension of the paper is added with partially backlogging or complete backlogging.

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