

Sierpinski Gasket Fractal Array Antenna

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ABSTRACT

A fractal is a recursively generated object having a fractional dimension. Many objects, including antennas, can be designed using the recursive nature of a fractal. This paper describes the use of fractal arrangements for the design of planar antenna arrays. Iterated Function System (IFS) is used to generate the fractal array (Sierpinski gasket). Once these antenna arrays are generated, the resulting radiated field is calculated using scripts written in MATLAB®. Comparison of two-dimensional planar arrays is done in detail.

Keywords: Fractal Array Antenna, Sierpinski Gasket, Random Arrays, Planar Arrays.

1. INTRODUCTION

In recent years, the science of fractal geometry has grown into a vast area of knowledge, with almost all branches of science and engineering gaining from the new insights it has provided. Fractal geometry is concerned with the properties of fractal objects, usually simply known as fractals.

It has been an intriguing question among the electromagnetic community [1] as to what property of fractals, if any, is really useful, especially when it comes to designing fractal shaped antenna elements. Several fractal geometries have been explored for antennas with special characteristics, in the context of both antenna elements and spatial distribution functions for elements in antenna arrays. Yet there have been no concerted efforts to design antennas using fractal shaped elements.

In many fractal antennas, the self-similarity and plane-filling nature of fractal geometries are often qualitatively linked to its frequency characteristics. The fractal geometry has been behind an enormous change in the way scientists and engineers perceive, and subsequently model, the world in which we live [2].

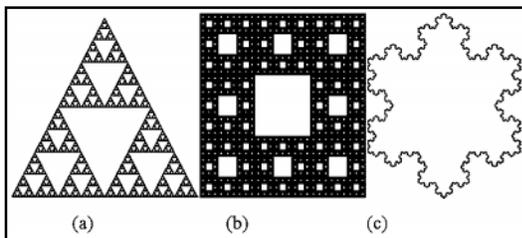


Fig.1: (a) Sierpinski Gasket (b) Sierpinski Carpet (c) Koch Snowflake Examples of Fractal Geometry for Antenna Designs

There are many fractal geometries that have been found to be useful in developing new and innovative

design for antennas, e.g. Sierpinski carpet, Cantor set, Koch curves, Sierpinski gasket, Koch snowflake.

2. FRACTAL BASICS

Fractals may be found in nature or generated using a mathematical recipe. The word “fractal” was coined by Benoit Mandelbrot, sometimes referred to as the father of fractal geometry, who said, “I coined fractal from the Latin adjective fractus. The corresponding Latin verb frangere means ‘to break’ to create irregular fragments. It is therefore sensible—and how appropriate for our need!—that, in addition to ‘fragmented’ (as in fraction or refraction), fractus should also mean ‘irregular’, both meanings being preserved in fragment” [3]. Moreover he asked: “Why geometry is often described as ‘cold’ or ‘dry’? One reason lies in its inability to describe the shape of a cloud, a mountain, a coastline, or a tree. Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line.”

To date, there exists no watertight definition of a fractal object. Mandelbrot offered the following definition: “A fractal is by definition a set for which the Hausdorff dimension strictly exceeds the topological dimension,” which he later retracted and replaced with: “A fractal is a shape made of parts similar to the whole in some way.” So, possibly the simplest way to define a fractal is as an object that appears self-similar under varying degrees of magnification, and in effect, possessing symmetry across scale, with each small part of the object replicating the structure of the whole. This is perhaps the loosest of definitions; however, it captures the essential, defining characteristic, that of self-similarity. Approximate fractals are easily found in

nature. Examples include clouds, snow flakes, crystals, mountain ranges, lightning, river networks, cauliflower or broccoli, and systems of blood vessels and pulmonary vessels. Coastlines may be loosely considered fractal in nature.

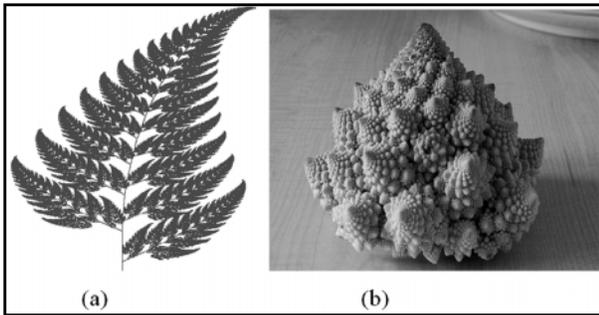


Fig.2: (a) Barnsley's Fern (b) Romanesco Broccoli an Example of Geometry Found in Nature that can be Easily Modelled using Fractals

3. SIERPINSKI GASKET GEOMETRY

Sierpinski gasket geometry is the mostly widely studied fractal geometry for antenna applications. Sierpinski gaskets have been investigated extensively for monopole and dipole antenna configurations. The self-similar current distribution on these antennas is expected to cause its multi-band characteristics [4]. It has been found that by perturbing the geometry the multi-band nature of these antennas can be controlled. Variations of the flare angle of these geometries have also been explored to change the band characteristics of antenna. Antennas using this geometry have their performance closely linked to conventional bow-tie antennas. However some minor differences can be noticed in their performance characteristics. It has been found that the multi-band nature of the antenna can be transformed into wideband characteristics by using a very high dielectric constant substrate and suitable absorbing materials.



Fig. 3: Sierpinski Gasket Evolution

4. CHARACTERIZATION OF OPTIMIZED ANTENNA ARRAYS

An optimized antenna would have no side lobes. The benefit of having only one beam is apparent when we consider airports. If the antenna array with radiation patterns shown in Fig 4(a) were used in an airport, the side lobes would easily cause air traffic control to confuse a large airplane at the height of the side lobes with a small plane at the peak of the main beam. Another characteristic of an optimized beam involves a thin single main beam [6]. Fig 4(b) shows the catastrophe that could

happen if a thick main beam were to confuse two airplanes as one large plane.

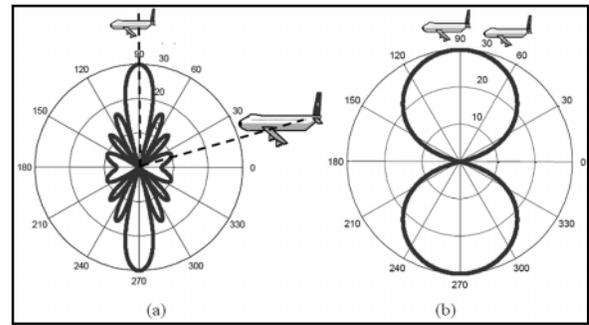


Fig.4: (a) Radiation Pattern with High Side Lobes (b) Radiation Patterns with Thick Main Beam

5. PLANAR ARRAYS

Two-dimensional antenna arrays allow for more flexibility and variety in placements of array elements. Two main ways in which elements have been placed on planar arrays are to place them on a grid or to randomly scattering them about a certain area. Although both still cause side lobes to exist, both arrangements have their advantages. We used MATLAB® to calculate the constructive and destructive interference in terms of the array factor (AF), that characterizes the radiated field. Array Factor (AF) for planar arrays is given as

$$AF = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} e^{-i2\pi \times M \left(\frac{d}{\lambda}\right) \cos \theta \sin \phi_c - i2\pi \times N \left(\frac{d}{\lambda}\right) \cos \theta \cos \phi}$$

To simplify the operation let $f_x = \frac{x}{\lambda r}$ and $f_y = \frac{y}{\lambda r}$ so that AF becomes:

$$AF = \sum_{n=0}^{N-1} e^{-i2\pi(f_x x + f_y y)} \tag{2}$$

6. PERIODIC ARRAYS

Planar arrays whose elements are organized in a grid have tendencies to produce main beams and side lobes of the same height. In Fig 5(a) 324 elements have been placed on a rectangular grid of 1.5x2 square units. These dimensions are chosen for scaling purposes, as our fractal array (discussed later) uses this size window. Fig 5(b) shows the Radiated field in a 3D where dark blue is the lowest point and red is the highest point.

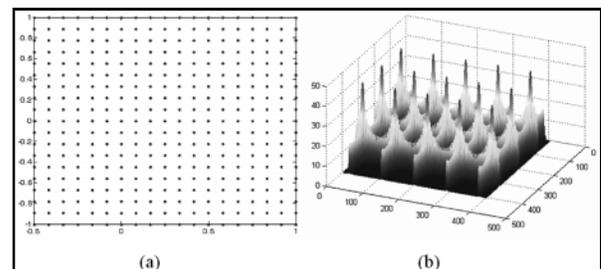


Fig.5: (a) 324 Elements with Equal x & y Distances Apart (b) Corresponding Radiated Field

7. RANDOM ARRAYS

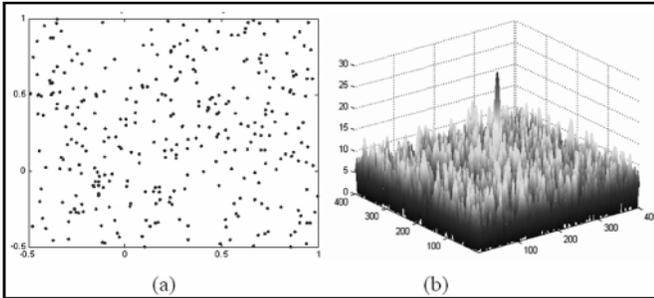


Fig.6: (a) 324 Elements Randomly Placed (b) Corresponding Radiated Field

Random planar arrays have radiation characteristics that may be more desirable. We plotted 324 elements on the same rectangular area of 1.5×2 square units. Fig 6(b) shows how side lobes are generally lower than the ones seen in the first set of ordered points. There is also an 180° symmetry, which is more apparent about the main beam.

8. FRACTAL ARRAYS

As the goal of our work is to characterize an optimized antenna array, we propose that the radiation properties of both the ordered fractals and the random fractals have redeeming qualities. Yet, there continues to be a gap between the ordered and the disordered. Fractals attempt to bridge that gap where they are constructed in an organized pattern, yet they also attain the qualities of random points depending on their construction and orientation of points. To first generate the fractals, we used an Iterated Function System (IFS) in the Matlab script that used probability to randomly place points within initial matrix boundaries. These initial matrix boundaries are specific to the Sierpinski gasket. For the sake of comparison and scaling, we also input the gasket to comprise only 324 points. Fig 7 shows the organization of the points and the characteristic radiation properties of this particular organization.

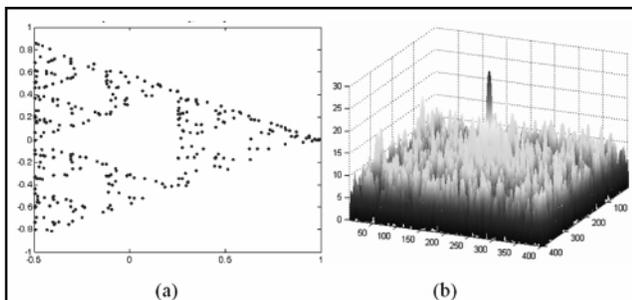


Fig.7: (a) 324 Elements Random Point Generated Sierpinski Gasket (b) Corresponding Radiated Field

Array theory explains the lines that can be seen extending from the middle point. For every "line" formed by the antenna elements, the radiation will have higher side lobes on a line perpendicular to the respective lines

formed. We observed this property earlier when we plotted the array factor for the points that were in a grid. Because the elements formed lines in both the x and y axes, there were lines of higher side lobes that overlapped, producing the grid-like organization of side lobes with equal heights. This is also why the random arrays are so effective in lowering the side lobes. Because the random placing of the elements on the array does not allow for "lines" to be formed, no perpendicular side lobe line is formed.

9. CONCLUSION

The qualities of an optimized antenna array deals mainly with the quality of the main beam, and the side lobe levels. As seen in the periodic array, the main beam was good in that there was no interference from smaller side lobes coming from the side. This main beam degradation can be seen in the random array where the main beam no longer maintains its point-like appearance on our graphs. The main beam characteristics of the periodic array were imitated by the fractal array [5].

Side lobe levels for periodic elements are higher yet farther apart because of large amounts of elements. The random array achieves lower side lobes with significantly less elements. Like the random array, the fractal had lower side lobes at lower stages of growth. Fig 8, fig. 9 and fig. 10 compares the side lobe levels of the periodic array, random arrays and fractal array.

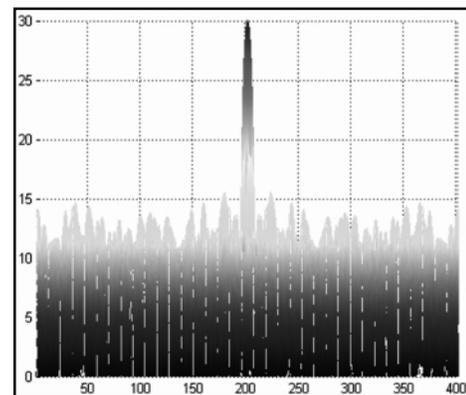


Fig.8: Side View of Radiated Field for Random Array

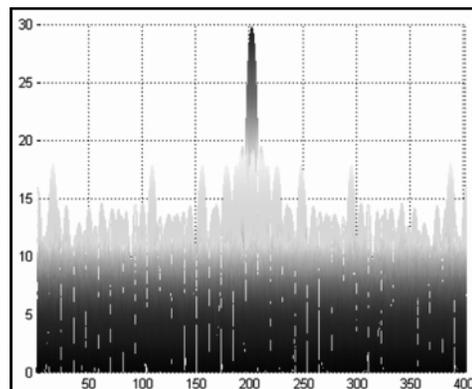


Fig.9: Side View of Radiated Field for Fractal Array

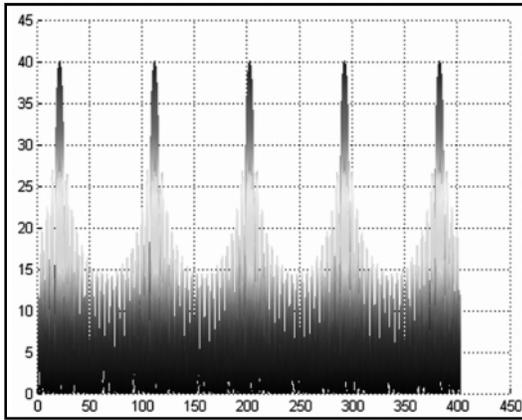


Fig.10: Side View of Radiated Field for Periodic Array

It can be observed that the random array performed better than the fractal array. According to previous works, there is an intersection point where the number of elements in a fractal array and random array cause for the one to be more effective as the other. As the array holds more elements, the random arrays perform better. Yet before this intersection point we have described, certain fractal arrays perform much better. Aside from the side lobe levels, the fractal did have overall a better main beam, regardless of the number of points. A drawback for random arrays with many elements at that, as the number of elements increases, main beam degradation is quite significant. The main beam completely abandons its point-like form.

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