

## Deadtime Modeling for First Order Plus Dead Time Process in a Process Industry

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### ABSTRACT

In this present work consistency of thick stock varying between 2.5%-3.5% in the wet end machine has been chosen for its operation and control. Normally, it gives a first order with dead time response and therefore it is an important candidate for performing the modeling of dead time for further analysis. The dead time expression involves an exponential term in the transfer function and to handle dead time in Laplace domain, it is customary to use approximations for the dead time expression. The Pade approximation is very useful in control analysis in this aspect. In this paper, dead time modeling analysis for an important parameter of paper mill, i.e. consistency is performed using MATLAB software with unit step response for different orders of Pade approximations viz first, fifth, tenth, fifteenth and twentieth order. The response results are analyzed, compared and important conclusions are derived. Further, it is shown that the higher the order of the approximation used, the more accurate it is at higher frequencies, but at the expense of increased complexity.

*Keywords:* Pade Approximation, Consistency, IMC, Dead Time

### 1. INTRODUCTION

Many pulp and paper processes are represented by first order plus dead time for tuning purposes. A PID controller can be used to control this type of process if the dead time is less than three times the process time constant. The regulatory control performance of the loop i.e. disturbance rejection, deteriorates rapidly when dead time exceeds time constant of the process model, even though the response to set point changes remains acceptable.

The first order plus dead time transfer function is defined as follows:

$$G_p(s) = \frac{K_p e^{-\theta_D s}}{1 + \tau s}; \text{ where } \theta_D \text{ is the dead time and } \tau \text{ is}$$

the time constant.

The integral gain remains constant as deadtime increases, but the proportional gain decreases. The proposed model is based on unsteady state material balance or energy balance or combination of both. The consistency control loop can be designed by various configurations such as negative feedback, cascade, feed forward and feedback combination, feed forward and cascaded feedback, ratio control. For simplicity negative feedback control configuration has been considered. Usually, the dilution water from various sources is always added to the thick stock immediately before fan pump and then led to flow to a consistency sensor, and then to the other equipments of approach flow system

including headbox. A feedback signal is obtained from the consistency sensor which is transmitted to the consistency controller through transmitter.

A comparator is used in the loop to compare the set point and measured variable to produce an error which goes to the controller to determine an appropriate position of the valve controlling the flow of dilution water to the stock immediately ahead of the pump. The value of the dead time for consistency control depends upon type of the process, loop design and location of sensor.

### 2. STEP RESPONSE

The transfer function of consistency control process can be adequately represented by first order plus dead time as under

$$G_p(s) = K_p [e^{-\theta_D s} / (1 + \xi s)]$$

Carrying out bump test on the approach flow system flow loop, Nancy developed the following dynamics equation with dead time of the order of 6.84s due to transmitter location relative to the dilution point. The time constant of 3.84s is due to the sensor measurement dynamics.

$$G_p(s) = -0.0407 e^{-6.84s} / (1 + 3.84s)$$

The above eqn. must be expressed in terms of effective process gain,  $K_{p(\text{effective})}$ . The effective process gain is defined as the ratio of % consistency and % controller output (2% to 4%). Using these values, we get the

following effective process transfer function with effective process gain of the order of -2.035.

$$G_p(s) = -2.035 e^{-6.84}/(1 + 3.84s)$$

The step response of consistency parameter is shown in figure 1.

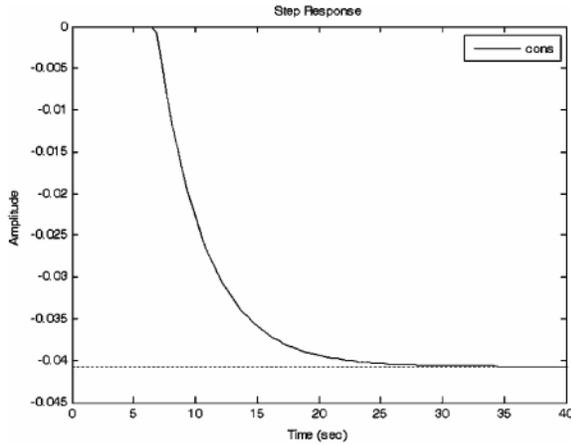


Fig. 1: Step Response of Consistency Transfer Function

3. DEAD-TIME MODELING

The Padé approximation creates a rational transfer function in the following way:

$$e^{-s\theta_D} = \frac{e^{(-s\theta_D/2)}}{e^{(s\theta_D/2)}} \approx \frac{1 + (-s\theta_D/2) + \frac{(-s\theta_D/2)^2}{2!} + \dots}{1 + (s\theta_D/2) + \frac{(s\theta_D/2)^2}{2!} + \dots}$$

The first-order Padé approximation is :

$$e^{-s\theta_D} \approx \frac{1 - s\theta_D/2}{1 + s\theta_D/2}$$

Applying this approximation to the consistency transfer function equation, we get:

$$G_p(s) = \frac{-0.0407 (1 - 3.42s)}{(3.8s + 1) (1 + 3.42s)} = \frac{-0.0407 (-3.42s + 1)}{(3.84s + 1) (3.42s + 1)}$$

As shown in figure 2, the response initially goes the wrong way due to the zero in the right-half plane and that it is fairly inaccurate during the early stages. By implication, it is inaccurate in the high frequency region and accurate in the low frequency region. In contrast, a fifth-order Padé approximation for the same transfer function is as follows:

$$G_p(s) = \frac{-0.0407 (-0.495s^5 + 2.17s^4 - 4.44s^3 + 5.20s^2 - 3.42s + 1)}{(3.84s + 1) (0.495s^5 + 2.17s^4 + 4.44s^3 + 5.20s^2 + 3.42s + 1)}$$

Due to presence of many more terms, this approximation should be more accurate for higher frequencies, than the first order Padé approximation.

Clearly, this is a more accurate approximation. The response oscillates several times during the deadtime

period by a small amplitude, thus giving a good approximation to deadtime. On the other hand, the expression for the transfer function is quite complex and is not as useful for algebraic manipulation as is the first-order approximation.

Figures 3, 4 and 5 shows the important comparisons of different orders of Padé approximations.

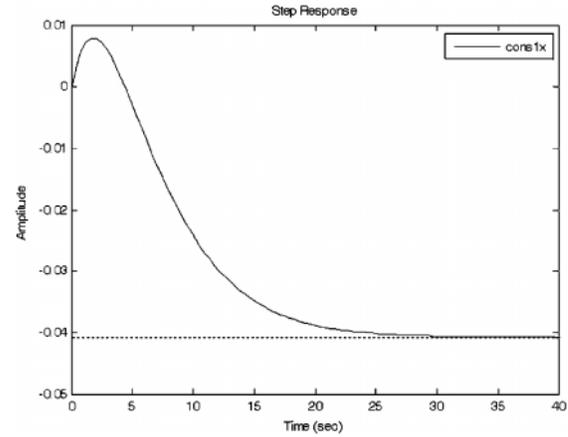


Fig. 2: Step Response for First Order Padé Approximation of Consistency Transfer Function

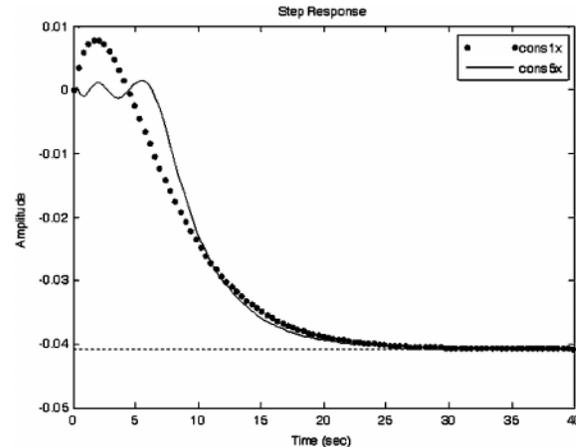


Fig. 3: Comparison between Step Response for First Order Padé and Fifth Order Approximation of Consistency Transfer Function

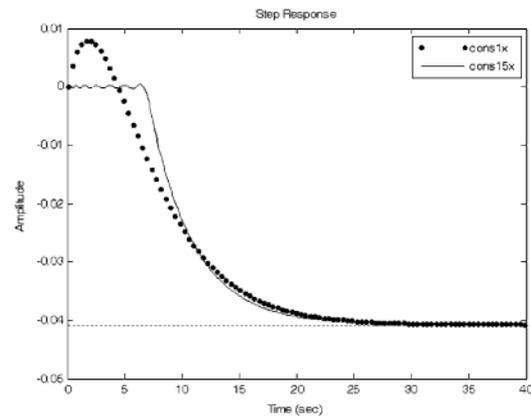


Fig. 4: Comparison between Step Response for First Order Padé and Fifteenth Order Approximation of Consistency Transfer Function

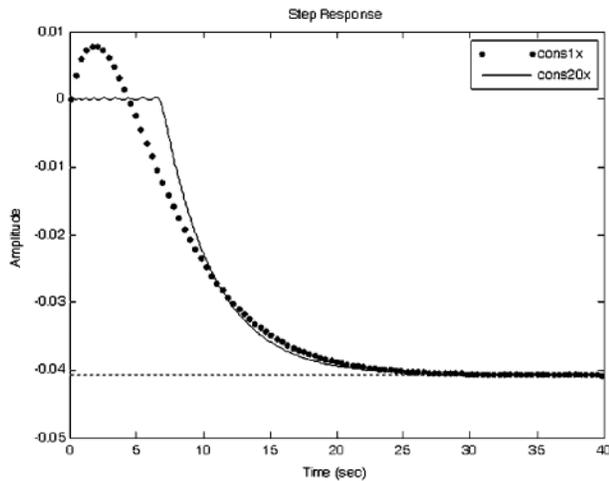


Fig. 5: Comparison between Step Response for First Order Padé and Twentieth Order Approximation of Consistency Transfer Function

## 5. CONCLUSION

The Padé's approximation of first order, second order or higher can provide a rational transfer function for various situations. The Padé approximation is very useful in control analysis. The first-order Padé approximation is useful because it is simple and is more accurate at higher frequencies. The higher the order of the approximation used, the more accurate it is at higher frequencies. The trade-off, however, is that as the mathematical expression

becomes more complicated, it is more difficult to gain insight from the expression, even though the numerical results are more accurate.

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