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AN ALTERNATIVE METHOD FOR OBTAINING INITIAL FEASIBLE SOLUTION TO A TRANSPORTATION PROBLEM AND TEST FOR OPTIMALITY

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ABSTRACT

The problem of finding the initial basic feasible solution of the Transportation Problem has long been studied and is well known to the research scholars of the field. In the simplex method degeneracy does not cause any serious difficulty whereas it usually causes computational problem in transportation technique. We also attempted to solve the transportation problem successfully and concluded that one short-cut approach to solve transportation problem is Vogel's Approximation Methos (VAM). But even in this method the degeneracy puts an obstacle at initial stage or at later stage. In past Sharma, S.D. has also successfully avoided the problem of degeneracy while solving for optimal solution of the transportation problem.

In this article, more efficient method for finding Initial Feasible Solution of the transportation problem has been developed. This method is based on min-max (max-min) criteria of well known Game Theory of Operations Research. The degeneracy problem has also been avoided by this method and therefore we claim that this method is more efficient than any other known method including Vogel's Approximation Method (VAM).

It is also to be noticed that this method requires lesser number of iterations to reach optimality as compared to other known methods for solving the transportation problem and the solution obtained is as good as obtained by Vogel's Approximation Method (VAM).

Keywords: Degeneracy, Non-degenerate Solution, Feasible Solution, Vogel's Approximation Method, Optimality, Min-max (max-min) Criteria.

1. INTRODUCTION

Degeneracy always puts an obstacle at initial or later stage when any transportation problem is solved for optimality. In past Sharma, S.D. has also successfully avoided the problem of degeneracy while solving for optimal solution of the transportation problem. We further pursued the problem and obtained the desired result.

In this article, an alternate technique for finding Initial Feasible Solution of the transportation problem has been developed. This method is based on min-max (max-min) criteria of Game Theory of Operations Research. The new technique requires lesser number of iterations to reach optimality as compared to other known methods for solving the transportation problem. The degeneracy problem is also avoided by this method and therefore it is better than Vogel's Approximation Method (VAM) also.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

Let C_{ij} be the cost of transportation of one unit product from i^{th} origin to j^{th} destination, and x_{ij} be the amount to be shipped from i^{th} origin to j^{th} destination.

If is also assumed that total availabilities $\sum a_i$; satisfy the total requirements $\sum b_j$, i.e.

$$\sum a_i = \sum b_j \quad (i=1,2,\ldots, m;\ j = 1,2,\ldots, n) \qquad (1)$$

The problem now is to determine non-negative values of 'x_{ij}' satisfying both the availability constraints

$$\sum_{j=1}^{n} x_{ij} = a_i; \quad \text{for } i = 1,2,\ldots, m \qquad (2)$$

and the requirement constraints

$$\sum_{j=1}^{n} x_{ij} = b_j; \quad \text{for } j = 1,2,\ldots, n \qquad (3)$$

and minimizing the total transportation cost

$$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} C_{ij} \qquad (4)$$

Let p = The minimum cost cell of the row in which the maximum cost C_{ij} lies.

i.e. $$p = \min[\max_i(C_{ij})] \qquad (5)$$

and q = The minimum cost cell of the column in which the maximum cost C_{ij} lies

i.e. $$q = \min[\max_j(C_{ij})] \quad (6)$$

Then the total transportation cost

$$TC = \sum p x_{ij} + \sum q x_{ij} \quad (7)$$

Algorithm

Step 1: First of all, we select the maximum cost cell (C_{ij})

Step 2: We move the corresponding row of the maximum cost cell (C_{ij}) and select the minimum cost cell.

Step 3: We allocate the maximum possible quantity to the minimum cost cell.

Step 4: Repeat steps (2) and (3) for the column cell containing the maximum (C_{ij}).

Step 5: In case of a tie, chose arbitrarily.

Step 6: Delete or cross out the row (column) when the allocation becomes complete. In case of a tie delete either row or column. If the row and column both deserve to be deleted put zero in the minimum cost cell of either row or column which are not yet deleted.

Step 7: Repeat step (2) to step (6) until all the allocation becomes complete.

3. ILLUSTRATIVE EXAMPLE

Let us consider a transportation problem

Table 1

From \ To	A	B	C	Supplies (a_i)
1	5	4	12	200
2	8	6	10	100
3	11	7	11	200
Demand(b_j)	100	200	200	500

Step 1: The maximum cost cell is C_{13}.

Step 2: We move the first row which contains C_{13} and select the cell C_{12} which is minimum in the first row.

Step 3: We allocate 200 to the cell C_{12} and cross out the second column and put zero in the cell C_{11} because first row is not deleted.

Step 4: Next we move in the third column and select the cell C_{23} which is minimum in that column and cross out the second row.

Step 5: We balance the remaining cells.

Table 2

From \ To	A	B	C	Supplies (a_i)
1	5 (0)	4 (200)	12	200
2	8	6	10 (100)	100
3	11 (100)	7	11 (100)	200
Demand(b_j)	100	200	200	500

I.F.S. = 5 × 0 + 4 × 200 + 10 × 100 + 11 × 100 + 11 × 100
= 4,000

Here total number of allocations = $m+n-1$ =3+3–1=5, therefore, it is non degenerate solution. Now we proceed to test whether the I.F.S. is optimal or not with the help of MODI Method.

Table 3

V_j	V_1 (11)	V_2 (10)	V_3 (11)		
From \ To	A	B	C	Supplies (a_i)	u_i
1	5 (0) +θ	4 (200) −θ	12	200	u_1 (-6)
2	8	6	10 (100)	100	u_2 (-1)
3	11 −θ (100)	7 +θ	11 (100)	200	u_3 (0)
Demand(b_j)	100	200	200	500	

$d_{13} = C_{13} - (u_1 + v_3)$
$= 12 - (-6+11) > 0$

$d_{21} = C_{21} - (u_2 + v_1)$
$= 8 - (-1+11) = -2 < 0$

$d_{22} = C_{22} - (u_2 + v_2)$
$= 6 - (-1+10) = -3 < 0$

$d_{32} = C_{32} - (u_3 + v_2)$
$= 7 - (0+10) = -3 < 0$

Since all d_{ij}'s are not positive. Therefore, it is not optimal solution. For finding the optimal solution we construct a loop with the help of cells (3,2), (3,1), (1,1) and (1,2). We allocate è to the cell (3,2).

∴ min (200 – θ, 100 – θ) = 0

∴ θ = 100

Now revised table is

Table 4

From \ To	A	B	C	Supplies (a_i)
1	5 (100)	4 (100)	12	200
2	8	6	10 (100)	100
3	11	7 (100)	11 (100)	200
Demand(b_j)	100	200	200	500

Total cost = 5 × 100 + 4 × 100 + 10 × 100 + 7 × 100 + 11 × 100
= 500 + 400 + 1000 + 700 + 1100
= 3700

It is also non degenerate solution. After applying MODI method, we find that it is optimal solution.

4. VERIFICATION OF THE METHOD BY VAM

Table 5

From \ To	A	B	C	Supplies (a_i)
1	5 (100)	4 (100)	12	200
2	8	6 (100)	10	100
3	11	7	11 (200)	200
Demand(b_j)	100	200	200	500

The initial basic feasible solution is degenerate since the number of used cells is 4 which is less than the number ($m + n - 1 = 3 + 3 - 1$) = 5. To deal with this problem we place a zero in the cell (3,2), then we apply the MODI method to test the optimality.

Table 6

V_j	V_1 (11)	V_2 (10)	V_3 (11)		
From \ To	**A**	**B**	**C**	**Supplies (a_i)**	**u_i**
1	5 (100)	4 (100)	12	200	u_1 (0)
2	8	6 (100)	10	100	u_2 (2)
3	11	7 (0)	11 (200)	200	u_3 (3)
Demand(b_j)	100	200	200	500	

Since all d_{ij}'s are non-negative, this solution is optimal. However, here $d_{23} = 0$. Therefore, we can seek the alternative choice for the cell to receive the zero.

Total cost = 100 × 5 + 100 × 4 + 100 × 6 + 200 × 11 + 7 × 0
= 3700

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