A Combination of Wavelet and Fractal Image Denoising Technique

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Abstract: In this paper, a new denoising technique for corrupted images with additive white Gaussian noise is presented. The technique used here is to combine the wavelet transform and the fractal transform with Recursive Gaussian diffusion. The experimental results obtained from various images show that this method gives better performance when compared to curvelet-based denoising and fractal-based denoising.

Keywords: Image Denoising, Wavelet Transform, Fractal Transform, Fractal Coding, Fractal Thresholding, Wavelet Thresholding.

1. INTRODUCTION

The recent advancement in multimedia technology has promoted an enormous amount of research in the area of image and video processing. Included in the many image and video processing applications, such as compression, enhancement, and target recognition, are preprocessing functions for noise removal. Noise removal is one of the most common and important processing steps in many image and video systems. Since their introduction two decades ago, wavelets have gained considerable interest in signal processing. The idea of representing a signal at multiple resolutions allows to capture its main trends in only a few coefficients while localizing discontinuities precisely. In the context of image processing, wavelets have been used for various applications such as denoising and compression, leading to standards such as JPEG2000. It is well known that wavelets are optimal for representing unidimensional (1-D) signals with a finite number of discontinuities [1], in the sense that the mean-squared error (MSE) of a nonlinear approximation from the maximal wavelet coefficients decreases in $O(k-1)$. In the case of images, for the sake of simplicity and efficiency, wavelets are often applied in a separable manner on the horizontal and vertical axis. This results in only a partial decorrelation of signal, which translates into clusters of highly energetic coefficients along the image contours. Although this residual dependency is reduced and partially captured by subband coders in the case of image compression, it might be of interest to have a transform which overcomes these drawbacks by filtering along the image contours.

Based on this observation, many oriented transforms have been designed recently. These transforms are either fixed, with each subband corresponding to a specific orientation and scale, or adaptive, with the direction of analysis chosen locally. Among the set of fixed transforms, the curvelet transform [2] provides nonlinear approximation close to the optimum without explicit knowledge of the image geometry. While this property is desired for compression, discrete versions of the curvelet transform are based on a finite set of blockwise Radon transforms [2] and highly overcomplete. Designed in the discrete domain, the contourlet transform [3], [4] provides a directional analysis similar to the curvelet transform with a reduced redundancy. This transform uses a fixed multiresolution directional filterbank based on iterated fan filters and a Laplacian pyramid. Using a carefully designed directional filterbank, the related CRISP contourlet approach [5] even achieves critical sampling. The steerable pyramid [6] and complex wavelet transform [7] are other examples of fixed directional transforms. While the redundancy of most of these transforms is not problematic for denoising applications, the increased number of coefficients at high rates compared to critically
sampled transforms is penalizing for compression applications.

Another problem seen in the image denoising techniques is the pseudo-Gibbs artifacts which occur due to the thresholding of some of the transform coefficients to zero. Sometimes artifacts cannot be permitted in result images. For example, in medical image processing, artifacts can bring mistake in diagnosis. These artifacts can be reduced by nonlinear diffusion [8]. Denoising by combining wavelet transform and anisotropic diffusion to reduce these artifacts has been explored before and shown to give positive results [9]. Recently, denoising by combining Fractlet shrinkage and non linear diffusion has been implemented and the results show significant reduction in the artifacts [10].

In this paper, we combine anisotropic diffusion along with our hybrid denoising method which involves the Fractlet transform and the wavelet transform. We also compare the results obtained by our method with those obtained by Fractlet-based denoising and wavelet based denoising methods.

2. FRACTAL TRANSFORM

Originally, fractal-based methods sought to express a target set as a union of shrunken copies of itself. However, most real-world images are rarely so entirely self-similar. Instead, self-similarity may be exhibited only locally, in the sense that subregions of an image may be self-similar. This is the basis of the block-based fractal encoding scheme introduced by Jacquin [11]. Most fractal-based image coding methods are based on this scheme, which can be outlined as follows:

1. As illustrated in Fig. 1, the image is subdivided into two different non-overlapping partitions of sub-blocks:
   
   (a) \( M \times M \) domain (parent) blocks, \( D_r \). For instance, when the image is square with a power of size, we choose \( M = 2^m \), for some integer \( m > 1 \).
   
   (b) \( N \times N \) domain (child) \( R_k \). Typically we choose \( N = 2 \times M \), so that the size of a parent block is four times of a child block.

2. Each child block, \( R_k \), \( k = 1, 2, 3 \ldots, N^2 \), is then matched to its most ‘similar’ parent block for the same.

3. The similarity between the parent block \( D_i \) \( (k) \) and the child block \( R_k \), is in the sense that the subimage on the parent block \( D_i \) \( (k) \) can be transformed “closely” to the subimage on the child block \( R_k \) via a contractive mapping. This contractive transformation is a composition of a geometric mapping \( w_i \) \( (k) \) followed by an affine grey-level mapping \( R_c \).

\[
\varphi (t) = \alpha t + \beta \quad \ldots \quad (1)
\]

2.1 Fractal Decoding

The fractal decoding algorithm is a fast and recursive process which can be summarized as follows: Starting with any initial image (typically a blank image), each range sub-block \( R_k \) is decoded from its fractal code. This process is then repeated recursively until a desirable convergence is achieved. Convergence may be defined in terms of the difference between two consecutive iteration estimates. Next, we outline the fractal-based joint image denoising and resizing scheme.

2.2 Fractal Image Denoising

In [12], a simple and effective fractal-based strategy for smoothing noisy images in the pixel as well as the wavelet domains of the noisy image was proposed. First, it was observed that straightforward fractal coding of a noisy image yields some degree of noise reduction. This may be explained by the fact that self-similar structures found in natural images are generally reconstructed rather well through fractal coding whereas the noisy components cannot be approximated well in this way. We have also shown that one can achieve better image denoising results by estimating the fractal code of the original noiseless image from that of the noisy observation. From this predicted fractal code, one can generate a fractally denoised estimate of the original image. Due to space limitation, the details of this fractal-based image denoising scheme are not included here. The reader is referred to our previous work in [13, 14].

2.3 Fractal Image Interpolation

In order to increase the size of the image by a power of two (in each direction), a simple fractal strategy can be applied [15]. This method is performed entirely during the fast converging fractal decoding process. For instance,ug for the commonly used test image of “Lena” of size 512 \( \times \) 512 and suppose we wish to double the size of this test image to obtain a fractal interpolated image of size 1024 \( \times \) 1024. (i) First, the original image is fractal encoded, using the standard fractal coding scheme, outlined above. This must be done at a sufficiently high fractal resolution (i.e. small domain/range blocks) in order to achieve a sufficiently high quality fractal representation of the image. For instance for the standard fractal scheme, we use \((M, N) = (64, 128)\) which results in mapping a 8 \( \times \) 8 pixel parent blocks into a 4 \( \times \) 4 child blocks. (ii) Then, in order to double the size of the fractally decoded

![Fig. 1: Uniform Image Partitioning for Fractal Image Coding.](image)
image, during the decoding process one simply starts with an initial blank image seed of the same size as the desired interpolated image. This simple fractal-based process can result in a fractally interpolated image of size 1024 × 1024 of course, modifying (increase or decreasing) the size of the image by any power of 2 in each direction can be performed in a similar manner, by simply applying the fractal code on an image blank image seed of a suitable size.

3. COMBINED DENOISING METHOD

In this combined denoising method, the noisy image is denoised using both the curvelet transform and the wavelet transform. The thresholding used is hard thresholding and undecimated wavelet transform is used for wavelet denoising. Anisotropic diffusion is then applied to both the denoised images. The resulting images are then combined by image fusion.

3.1 Fractal Thresholding

Fractal-wavelet (FW) transforms, discovered independently by a number of researchers (16-17), and [18] to name only a few, were introduced in an effort to reduce the blockiness and computational complexity that are inherent in fractal image compression. Their action involves a scaling and copying of wavelet coefficient subtrees to lower subtrees, quite analogous to the action of fractal image coders in the spatial domain.

Recall that the discrete wavelet transform (DWT) coefficients of a two-dimensional (2-D) signal (image) are conveniently arranged in a matrix, the first few blocks of which are shown in Fig. 1 (here, we are assuming that the 2-D wavelet basis functions are constructed in the usual way by using suitable tensor products of one-dimensional scaling and wavelet functions). The three collections of blocks comprise the fundamental horizontal, vertical, and diagonal quadrants of the coefficient tree.

In the standard FW scheme (as proposed in [5] and [12], for example) it is assumed that common parents and scaling factors are used for the three fundamental subbands. Of course, such a restriction is performed at the expense of image fidelity. Historically, it was employed for purposes of data compression. For FW decoding, the usual procedure is to start with a wavelet coefficient tree that contains the stored wavelet coefficients and zeros elsewhere. Iteration of the fractal-wavelet scaling and copying procedure produces a fixed point wavelet coefficient matrix that is an approximation to that of the original image. The smaller the collage distance in (1), the better the approximation [15].

Strictly speaking, the above expressions are approximations to the statistical quantities of the random variables and since they represent limited sample statistics. The fact that will not be very large in our applications will contribute to errors in estimating the local image statistics and, subsequently, suboptimal fractal codes for the noiseless images.

$$\alpha^* = \frac{E[XY]}{E[X^2]} \quad \ldots (2)$$

where

$$E[XY] = \frac{1}{n} \sum_{m=1}^{\infty} x_m y_m$$ and $$E[X^2] = \frac{1}{n} \sum_{m=1}^{\infty} x_m^2 \quad \ldots (3)$$

3.2 Wavelet Thresholding

Wavelet thresholding is a simple non-linear technique, which operates on one wavelet coefficient at a time. In its most basic form, each coefficient is threshold by comparing against threshold, if the coefficient is smaller than threshold, set to zero; otherwise it is kept or modified. Replacing the small noisy coefficients by zero and inverse wavelet transform on the result may lead to re-construction with the essential signal characteristics and with less noise. Wavelet thresholding involves three steps: A linear forward wavelet transform, non-linear thresholding step and a linear inverse wavelet transform. Let us consider a signal \( \{ x_{ij}, i, j = 1, 2, \ldots, M \} \) denote the \( M \times M \) matrix of the original image to be recovered and \( M \) is some integer power of 2. During transmission the signal is corrupted by independent and identically distributed (i.i.d) zero mean, white Gaussian Noise \( Z_{ij} \) with standard deviation \( \sigma \).

$$y_{ij} = x_{ij} + z_{ij} \quad \ldots (4)$$

From this noisy signal \( y_{ij} \), we want to find an approximation \( \tilde{x}_{ij} \). The goal is to estimate the signal \( \tilde{x}_{ij} \) from noisy observations \( y_{ij} \) such that Mean Squared Error (MSE) is minimum

$$\|X - \tilde{X}\|^2 = \frac{1}{N} \sum_{i=0}^{N-1} |x_i - \tilde{x}_i|^2 \quad \ldots (5)$$

Let \( W \) and \( W^{-1} \) denote the two-dimensional orthogonal discrete wavelet transform (DWT) matrix and its inverse respectively. Then Eq. (3) can be written as

$$\alpha_{ij} = x_{ij} + z_{ij} \quad \ldots (6)$$

With \( d = W^* y, c = W^* x, e = W^* z \). Since \( W \) is orthogonal transform, \( e = W^* z \) is also an i.i.d Gaussian random variable. The wavelet thresholding function then the wavelet thresholding based Denoising scheme can be expressed as

$$\tilde{x} = W^{-1} (T(Wy)) \quad \ldots (7)$$

Wavelet transform of noisy signal should be taken first and then thresholding function is applied on it. Finally the output should be undergone inverse wavelet transformation to obtain the estimate \( x \). There are four thresholds frequently used, i.e. hard threshold, soft threshold, semi-soft threshold, and semi-hard threshold. The hard-thresholding function
keeps the input if it is larger than the threshold; otherwise, it is set to zero. It is described as
\[
f_h (x) = \begin{cases} x & \text{if } x \geq \lambda \\ 0 & \text{otherwise} \end{cases}
\]

The hard-thresholding function chooses all wavelet coefficients that are greater than the given threshold sets the others to zero. The threshold is chosen according to the signal energy and the noise variance \( \sigma^2 \). If a wavelet coefficient is greater than, assumes that as significant and attribute it to the original signal. Otherwise, consider it to be due to the additive noise and discard the value. The soft-thresholding function has a somewhat different rule from the hard-thresholding function. It shrinks the wavelet coefficients towards zero, which is the reason why it is also called the wavelet shrinkage function. The soft-thresholding rule is chosen over hard-thresholding, for the soft-thresholding method yields more visually pleasant images over hard thresholding.

### 3.3 Gaussian Diffusion

Image smoothing using nonlinear diffusion has been used in the field of image processing. If \( u_0 \) denote the observed noisy image which is known to be the sum of the original images and some Gaussian noise \( n \)

\[
u_0 (x) = S (x) + p (x) - 2q (x), x = (x_1, x_2) \in R^2 \quad \ldots \quad (8)
\]

We consider the diffusion process

\[
\frac{\partial u}{\partial t} = \Delta \cdot (g (|\Delta u|) \Delta u)
\]

\( \ldots \quad (9) \]

with the given noisy signal \( u_0 \) as the initial condition

\[
u (x, 0) = u_0 (x), x \in R^2
\]

\( \ldots \quad (10) \]

and periodic boundary conditions. Here, the time \( t \) acts as a scale parameter for filtering. The choice of the diffusion constant \( g (x) = \text{const} \) corresponds to a strong smoothing of \( u \) with increasing \( t \). Typically, \( g (x) \) is a non-negative decreasing function with limit of \( g (x) \) tending to zero as \( x \) approaches infinity. The diffusivity \( g \) controls the smoothing process by admitting strong diffusion if the gradient is small, possibly caused by noise, and by slowing down or even stop the smoothing for large gradients. One of the serious problems in the diffusion model in (5) is that it is very sensitive to noise. The noise often introduces very large oscillations of the gradient. Therefore, the model in (4) possibly misconstrues the true edges and heavy noise, which leads to undesirable diffusion in regions where there is no true edge. A typical improved model, to remedy these deficiencies, is based on Gaussian regularization

\[
\frac{\partial u}{\partial t} = \Delta \cdot (g (\Delta (G\sigma * u)) \Delta u - \Delta p)
\]

\( \ldots \quad (11) \]

where \( G\sigma \) is a Gaussian kernel with variance. The Gaussian filtering acts as a preprocessing to reduce the influence of noise during the diffusion process. The diffusion is applied to the difference image \( u_0 - u_c \), instead of directly applying to the denoised image. Here \( u_0 \) is the noisy image and \( u_c \) is the denoised image. After some iteration steps of the diffusion scheme, the smoothed difference image is added to the denoised image to obtain the final result. The idea here is to keep the low-frequency coefficients and the significant coefficients, that is those coefficients whose magnitude is above the threshold, untouched and slightly change the coefficients that have been thresholded to zero in such a way that the image is smoothed.

The idea behind this approach is that while the denoised image contains the important features of the image to be reconstructed, the difference image particularly contains high-frequency components which mainly correspond to noise. Applying the diffusion only to the difference image avoids the narrow peaks or textures from getting smoothened too much as in conventional diffusion. We can thus retain the signal amplitude of the detail components while reducing the pseudo-Gibbs oscillations at the same time, in comparison to those methods directly using curvelet shrinkage. As the diffusivity function \( g \).

### 4. Simulation Results

For our experiments, we selected some 512 by 512 grayscale images with edges and some images with both edges and smooth regions. The images were corrupted with additive white Gaussian noise of different variance and the denoising method was applied. The noisy image was filtered using hard thresholding of the transform coefficients. For wavelet-based denoising, undecimated wavelet denoising was performed using the Daubechies-Antonini 7/9 filters and hard thresholding of the wavelet coefficients. The performance metric used are peak signal-to-noise ratio (PSNR) and visual quality of the denoised image. PSNR is given by,

\[
PSNR = 10 \log \left( \frac{p - p \text{ value of the image}}{MSE} \right)^2
\]

\( \ldots \quad (12) \]

4.1 Image Denoising Experiment

Experiments on the 256 × 256 noisy House image are used to validate the proposed edge detection scheme. The Canny edge detection is employed for comparison. In the following experiments, we fixed the small scale as 22 and then the large scale is 23. The variances are normalized to be equal to 1. Figure 2 (a) is noisy House image added Gaussian noise with the standard deviation is 0.015. Compared with (b) and (c),
it's evident that canny operators are sensitive to noise and eliminate some detail edges. Nevertheless, the edge image detected by the scale multiplication scheme has an excellent noise rejection capability and precise localization with clear edge profile.

4.2 Combined Denoising Technique

The test images Lena and House (256 × 256) have been corrupted with Gaussian noise and with different values of the standard deviation (0.1, 0.05, 0.32). Figure 3 shows the vision effect of two kind of denoising methods of image Lena. In Table 1 we give an objective estimation of the quality of the results, evaluated by means of the peak signal-to-noise ratio (PSNR) and the mean square error (MSE). The result indicated that edge details which deal with by traditional wavelet hard thresholding denoising method become more blurred as the threshold value increasing. The proposed method outperforms the traditional method, highlighting its particular effectiveness that presents sharp edges and many little details especially at a large threshold value. And the denoising parameters like peak signal-to-noise ratio are also better than the traditional wavelet denoising method.

Fig. 2: (a) Noisy House Image with \( \sigma = 0.015 \), (b) Gaussian Diffusion Map Detected, and (c) Combined Image Denoising Technique

Fig. 3: (a) Noisy Lena Image with \( \sigma = 0.001 \), (b) Denoised Image with the Proposed Algorithm, (c) Denoised Image with Wavelet De-noising, (d) Noisy Lena Image with \( \sigma = 0.025 \), (e) Denoised Image with the Proposed Algorithm, (f) Denoised Image with Traditional Wavelet Denoising, (g) Lena Noisy Image with \( \sigma = 0.22 \), (h) Denoised Image with the Proposed Algorithm, and (i) Denoised Image with Traditional Wavelet/Fractal Denoising.
5. CONCLUSIONS

The aim of this work was to shed some further insight into the fractal-based image denoising methods previously proposed in [5]. The essence of fractal-based denoising, both in the wavelet as well as pixel domains, is to predict the fractal code of a noiseless image from its noisy observation. We have experimentally shown that the fractal-wavelet denoising scheme is able, at least for moderate noise variances, to locate near-optimal parent subtrees that lie among the best domain subtrees in terms of collage distance. The procedure is assisted by the high degree of local self-similarity of an image. In general, in the pixel domain, a good number of domain subblocks approximate a given range subblock very well. And the fractal-based denoising method works well in finding one of these subblocks. This is also reflected in the wavelet domain. We have incorporated cycle spinning into these fractal-based denoising methods to produce enhanced estimates of the denoised image. We have also found that pixel-based fractal denoising schemes consistently perform slightly better, in terms of PSNR, than their wavelet-based counterparts (this does not, however, imply that the visual quality of the images is higher). Some possible explanations have been forwarded. However, the significant computational savings of the wavelet-based fractal denoising scheme may make it advantageous to use.

REFERENCES


