

# Some Aspects on Electromagnetic Scalar and Vector Potentials in 2D Space: A Review Paper

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**Abstract:** This paper presents some details on scalar and vector potential in electromagnetism and discusses some characteristics of integrability conditions, gravitational potential energy and pressure as buoyant potential. Brief introduction to electromagnetism and some fundamental equations of scalar and vector potential are also presented with the help of images and the graphs of scalar and vector potentials.

**Keywords:** Electromagnetism, Vector potential, Scalar potential, Magnetic field, Induction motors.

## 1. INTRODUCTION

Electromagnetism is the branch of science concerned with the forces that occur between electrically charged particles. There exists some electromagnetic force in the electromagnetism. This electromagnetic force is one of the four fundamental interactions in nature, the other three being the strong interaction, the weak interaction and gravitation. The word is a compound from two Greek terms, *ēlektron*, “amber” (as electrostatic phenomena were first described as properties of amber by the philosopher Thales), and *magnētēs*, “magnet” (the magnetic stones found in antiquity in the vicinity of the Greek city of Magnesia, in Lydia, Asia Minor).

Electromagnetism is the interaction responsible for almost all the phenomena encountered in daily life, with the exception of gravity. Ordinary matter takes its form as a result of intermolecular forces between individual molecules in matter. Electrons are bound by electromagnetic wave mechanics into orbitals around atomic nuclei to form atoms, which are the building blocks of molecules [1-2].

Electromagnetism manifests as both electric fields and magnetic fields. As shown in Fig. 1 and Figure 2, both fields are simply different aspects of electromagnetism, and hence are intrinsically related - a changing electric field generates a magnetic field; conversely a changing magnetic field generates an electric field. This effect is called electromagnetic induction, and is the basis of operation for electrical generators, induction motors, and transformers. The theoretical implications of electromagnetism led to the development of special relativity by Albert Einstein in 1905 [1,8-9].

Electric fields are the cause of several common phenomena, such as electric potential (such as the voltage of a battery) and electric current (such as the flow of electricity

through a flashlight). Magnetic fields are the cause of the force associated with magnets. In quantum electrodynamics, electromagnetic interactions between charged particles can be calculated using the method of Feynman diagrams, in which picture messenger particles called virtual photons being exchanged between charged particles. This method can be derived from the field picture through perturbation theory.

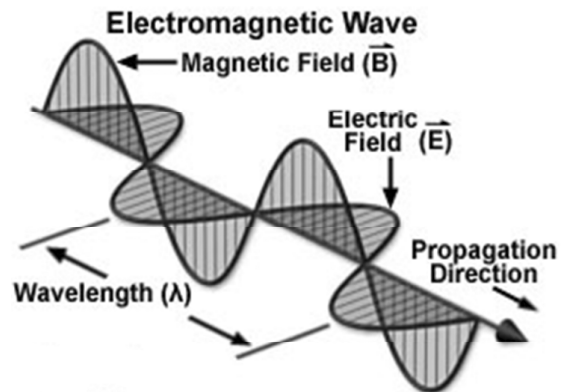


Figure 1: Electromagnetic wave

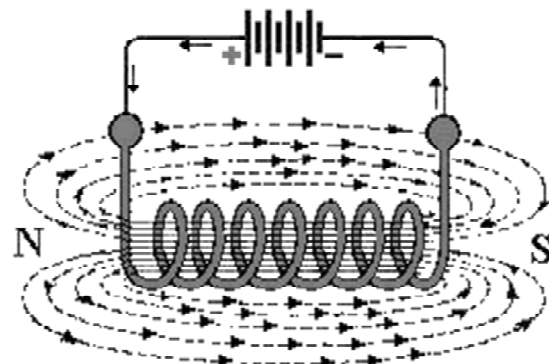


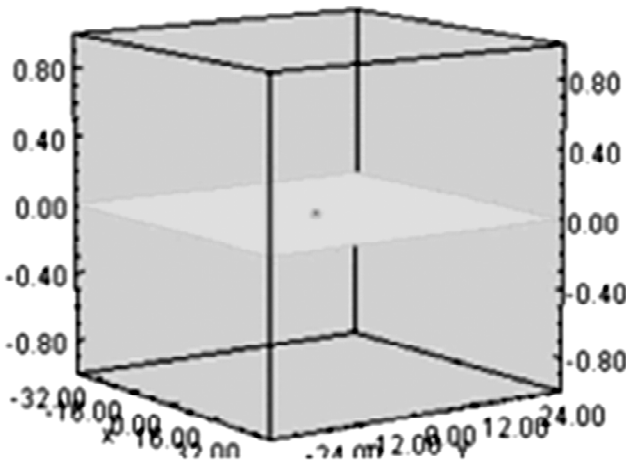
Figure 2: Electromagnetic Field Around a Coil

**2. ELECTROMAGNETIC PHENOMENA**

With the exception of gravitation, electromagnetic phenomena as described by quantum electrodynamics (which includes classical electrodynamics as a limiting case) account for almost all physical phenomena observable to the unaided human senses, including light and other electromagnetic radiation, all of chemistry, most of mechanics (excepting gravitation), and, of course, magnetism and electricity. Magnetic monopoles (and “Gilbert” dipoles) are not strictly electromagnetic phenomena, since in standard electromagnetism, magnetic fields are generated not by true “magnetic charge” but by currents. There are, however, condensed matter of magnetic monopoles in exotic materials (spin ice) created in the laboratory [1,3-4].

**3. SCALAR POTENTIAL**

Scalar potential describes the situation where the difference in the potential energies of an object in two different positions depends only on the positions, not upon the path taken by the object in travelling from one position to the other. It is a scalar field in three-space: a directionless value (scalar) that depends only on its location. Fig. 3 describes about the scalar gravitational potential well of an increasing mass in 3D space.



**Figure 3: Gravitational Potential well of an Increasing Mass where**

A scalar potential is a fundamental concept in vector analysis and physics (the adjective scalar is frequently omitted if there is no danger of confusion with vector potential). The scalar potential is an example of a scalar field. Given a vector field ( $F$ ), the scalar potential  $P$  is defined such that,

$$F = -\nabla P = \left( \frac{\partial P}{\partial x}, \frac{\partial P}{\partial y}, \frac{\partial P}{\partial z} \right)$$

Where,  $\nabla P$  is the gradient of  $P$  and the second part of the equation is minus the gradient for a function of the cartesian coordinates  $x,y,z$ . In some cases, we can use a

positive sign in front of the gradient to define the potential. Because of this definition of  $P$  in terms of the gradient, the direction of  $F$  at any point is the direction of the steepest decrease of  $P$  at that point, its magnitude is the rate of that decrease per unit length.

In order for  $F$  to be described in terms of a scalar potential only, the following have to be true:

$$-\int_a^b F \cdot dl = P(b) - P(a)$$

where the integration is over a Jordan arc passing from location  $a$  to location  $b$  and  $P(b)$  is  $P$  evaluated at location  $b$ .

$$\oint F \cdot dl = 0$$

where the integral is over any simple closed path, otherwise known as a Jordan curve.

$$\nabla \times F = 0$$

The first of these conditions represents the fundamental theorem of the gradient and is true for any vector field that is a gradient of a differentiable single valued scalar field  $P$ . The second condition is a requirement of  $F$  so that it can be expressed as the gradient of a scalar function. The third condition re-expresses the second condition in terms of the curl of  $F$  using the fundamental theorem of the curl. A vector field  $F$  that satisfies these conditions is said to be irrotational (Conservative) [5].

**4. VECTOR POTENTIAL**

In vector calculus, a vector potential is a vector field whose curl is a given vector field. This is analogous to a scalar potential, which is a scalar field whose gradient is a given vector field [5-6].

Formally, given a vector field  $v$ , a vector potential is a vector field  $A$  such that

$$v = \nabla \times A$$

If a vector field  $v$  admits a vector potential  $A$ , then from the equality

$$\nabla \cdot (\nabla \times A) = 0$$

**5. MAGNETIC VECTOR POTENTIAL**

Fig. 4 shows the graph of scalar and vector potential with magnetic field. The magnetic vector potential  $A$  is a vector field defined along with the electric potential  $\phi$  (a scalar field) by the equations:

$$B = \nabla \times A \quad E = -\nabla\phi - \frac{\partial A}{\partial t},$$

where  $B$  is the magnetic field and  $E$  is the electric field. Defining the electric and magnetic fields from potentials automatically satisfies two of Maxwell's equations: Gauss's law for magnetism and Faraday's Law.

$$\nabla \cdot A = \nabla \cdot (\nabla \times A) = 0$$

$$\nabla \times E = \nabla \times \left( -\nabla\phi - \frac{\partial A}{\partial t} \right) = -\frac{\partial}{\partial t} (\nabla \times A) = -\frac{\partial B}{\partial t}.$$

Alternatively, the existence of  $A$  and  $\phi$  is guaranteed from these two laws using the Helmholtz's theorem. For example, since the magnetic field is divergence-free (Gauss's law for magnetism), i.e.  $\nabla \cdot B = 0$ ,  $A$  always exists that satisfies the above definition. The vector potential  $A$  is used when studying the Lagrangian in classical mechanics and in quantum mechanics (see Schrödinger equation for charged particles, Dirac equation, Aharonov-Bohm effect) [7-10].

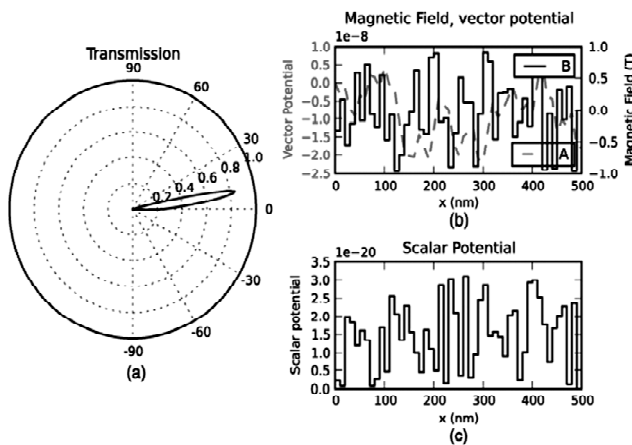


Figure 4: Graph Scalar and Vector Potential with Magnetic Field

6. INTEGRABILITY CONDITIONS

Consider  $F$  is a conservative vector field (also called irrotational, curl-free, or potential), and its components have continuous partial derivatives, the potential of  $F$  with respect to a reference point  $r_0$  is defined in terms of the line integral:

$$V(r) = -\int_c F(r) \cdot dr = -\int_a^b F(r(t)) \cdot r'(t) dt$$

where  $C$  is a parametrized path from  $r_0$  to  $r$

$$r(t), a \leq t \leq b, r(a) = r_0, r(b) = r.$$

The fact that the line integral depends on the path  $C$  only through its terminal points  $r_0$  and  $r$ , in essence, the path independence property of a conservative vector field [5].

7. ALTITUDE AS GRAVITATIONAL POTENTIAL ENERGY

Fig. 5 shows the uniform gravitational field near the Earth's surface and Fig. 6 depicts the plot of a two-dimensional slice of the gravitational potential in and around a uniform spherical body. The inflection points of the cross-section are at the surface of the body.

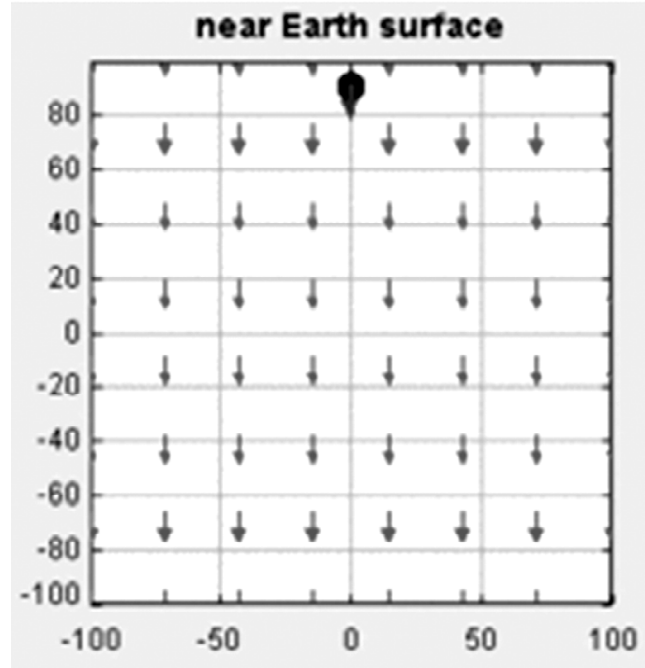


Figure 6: Uniform Gravitational Field Near the Earth's Surface

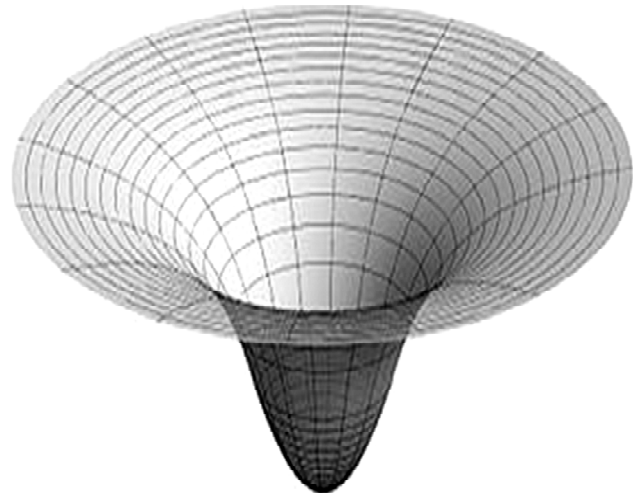


Figure 7: Plot of a 2D Slice of the Gravitational Potential in and Around a Uniform Spherical Body

The (nearly) uniform gravitational field near the Earth's surface has a potential energy which is given by,

$$U = mgh$$

where  $U$  is the gravitational potential energy and  $h$  is the height above the surface. This means that gravitational potential energy on a contour map is proportional to altitude. On a contour map, the two-dimensional negative gradient of the altitude is a two-dimensional vector field, whose vectors are always perpendicular to the contours and also perpendicular to the direction of gravity. But on the hilly region represented by the contour map, the three-dimensional

negative gradient of  $U$  always points straight downwards in the direction of gravity  $F$ . However, a ball rolling down a hill cannot move directly downwards due to the normal force of the hill's surface, which cancels out the component of gravity perpendicular to the hill's surface. The component of gravity that remains to move the ball is parallel to the surface

$$F_s = -mg \sin \theta$$

where  $\theta$  is the angle of inclination, and the component of  $F_s$  perpendicular to gravity is

$$F_P = -mg \sin \theta \cos \theta = -\frac{1}{2} mg \sin 2\theta$$

Let  $\Delta h$  be the uniform interval of altitude between contours on the contour map, and let  $\Delta x$  be the distance between two contours. Then

$$\theta = \tan^{-1} \frac{\Delta h}{\Delta x}$$

so that

$$F_P = -mg \frac{\Delta x \Delta h}{\Delta x^2 + \Delta h^2}.$$

However, on a contour map, the gradient is inversely proportional to  $\Delta_x$ , which is not similar to force  $F_P$ : altitude on a contour map is not exactly a two-dimensional potential field. The magnitudes of forces are different, but the directions of the forces are the same on a contour map as well as on the hilly region of the Earth's surface represented by the contour map [5].

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