

Analyzing Different PAPR Reduction Techniques Including MIMO OFDM System

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Abstract: In this paper, we propose a PAPR reduction method for OFDM system which is a challenging issue for OFDM system. In this paper we analyze different OFDM PAPR reduction techniques based on its computational complexity, bandwidth availability its BER and Power Consumption. We also analyze some methods of PAPR reduction for multiuser OFDM broadband communication system like MIMO-OFDM and SISO-OFDM. Simulation results shows highly improvement in PAPR relative to other OFDM techniques..

Index Terms-Peak average power ratio (PAPR),Orthogonal frequency division multiplexing (OFDM),Multiple input multiple output (MIMO),Single input single output (SISO).

I. INTRODUCTION

As the high data rate transfer, the orthogonal frequency division multiplexing (OFDM) is the most favourable technique in current broadband wireless communication system. The basic principle of OFDM is to divide a high-rate data stream into a number of lower rate data streams which are transmitted simultaneously over a number of subcarriers.

Since the carriers i.e. (sine and cosine)waves,we all know thatthe area under one portion of time of these are zero, means they are all orthogonal to each other. The property of orthogonal i.e.orthogonality which allows simultaneous transmission of sub carriers in a compact frequency space without interference from each other and it also allows high spectral efficiency.

It is familiar that OFDM is spectrally efficient not power efficient due to the large peak-to average power ratio (PAPR) inherent in the OFDM signals, a large value of PAPR also makes the signal worst or vulnerable to nonlinearities in the transmission.

There are number of approaches which have been proposed for reducing the PAPR of OFDM signals. For example, selective mapping or transformation [1], [2] may statistically reduce the PAPR with a relatively simple implementation cost.

This is the technique which has potential to reduce the PAPR. Systematic coding techniques may be attractive since they can deterministically bound the PAPR with little computational cost at the transmitter, but we have to design the low PAPR codes while maintaining a reasonable coding rate becomes quite difficult as the number of subcarriers increases.Mostly simplest for the PAPR reduction is digital clipping and filtering of the OFDM signal. The problem arises, however, that low pass filtering the clipped OFDM signal samples results in considerable

regrowth of peak power in addition to a certain amount of degradation in bit-error rate performance. In clipping technique, OFDM signal peaks larger than some threshold which is deliberately clipped off. Even though this is a simplest technique, it introduces several distortionslike in-band distortion and out of band noise.

In Selective Mapping (SLM) technique, by making different phase changes on identical input the sequence with the lowest PAPR is selected and transmitted. For the recovery of data, the receiver should have knowledge about the generation of OFDM signal and phase information. This information is said to be Side information (SI), which results in some loss of efficiency.

To mitigate the performance degradation in the propagation channel, channel coding is generally used in communication systems.

II. CHARACTERISTICS OF OFDM SIGNAL

Let Suppose a block of N symbols i.e. $X=\{X_K, K=0,1,\dots,N-1\}$ is formed with each symbol modulating one of a set of subcarriers $\{f_k,k=0,1,\dots,N-1\}$,where N is number of subcarriers. Here N subcarriers are chosen to be orthogonal, that is, $f_k=K\Delta f$, where $\Delta f=1/(NT)$ and T is original symbol period.

Therefore, the complex envelope of transmitted OFDM signal can be written as

$$x(t) = \frac{1}{\sqrt{N}} \sum_{K=0}^{N-1} X_K e^{j2\pi f_k t}, 0 \leq t \leq NT \quad (1)$$

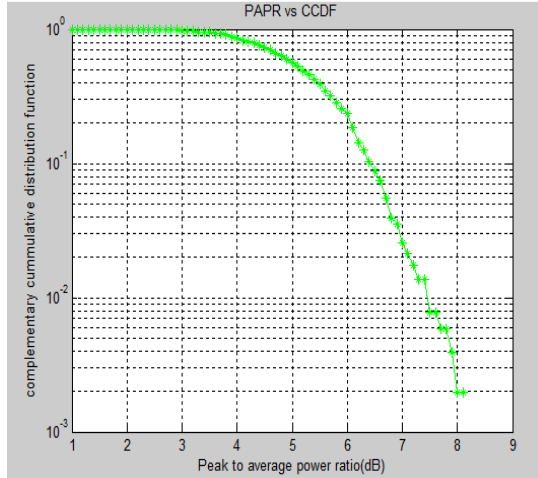


Fig.1. Distribution of PAPR of OFDM signal samples.

Where $j = \sqrt{-1}$.

III. PAPR MEANING IN OFDM

A data block of n symbols $X = [x_0, x_1, \dots, x_{n-1}]$ from signal constellation. The complex envelope $S_x(t)$ in OFDM system can be written as

$$S_x(t) = \sum_{k=0}^{n-1} x_k e^{\frac{j2\pi kt}{T}}, 0 \leq t \leq T \quad (2)$$

Where $1/T$ is the bandwidth of subcarrier.

The instantaneous envelope power $P_x(t)$ is defined by $P_x(t) = |S_x(t)|^2$ and its average power $\|X\|^2$ is

$$\|X\|^2 = \frac{1}{T} \int_0^T P_x(t) dt = \sum_{k=0}^{n-1} |x_k|^2 \quad (3)$$

Let X be the collection of all possible transmitted data blocks in OFDM system. The PAPR of transmitted data block $x \in X$ is defined as

$$PAPR(X) = \frac{\max_{0 \leq t \leq T} P_x(t)}{P_{av}} \quad (4)$$

Where P_{av} is average power of set X and $P(x)$ is the probability of X . We also define PAPR of a code X by

$$PAPR(X) = \max_{x \in X} PAPR(x) \quad (5)$$

IV. INDUCEMENT OF PAPR REDUCTION

A. Non-linear characteristic of ADC and HPA

Several radio stations have HPA in their transmitters to achieve sufficient transmission power at the transmitting end. To achieve maximum output power

efficiency, HPA is almost operated at or near saturation region.

In general, HPA requires a large back-off from the peak power to reduce the distortion which is caused by nonlinearity of HPA and this gives rise to a low power effectiveness, which is a significant load, especially immobile terminals. The maximum possible output power is limited by P_{in} when the corresponding input power is given by due to the aforementioned saturation characteristic of the amplifier. The input power must be backed off to operate in the linear region. The input-output characteristic of HPA consist of input back-off (IBO) and output back-off (OBO).

$$\text{Input Back-off} = 10 \log_{10} \frac{P_{in}^{max}}{P_{in}} \quad (6)$$

$$\text{Output Back-off} = 10 \log_{10} \frac{P_{out}^{max}}{P_{out}} \quad (7)$$

Due to high PAPR of OFDM signal, HPA introduce intermodulation between different subcarrier and also introduce additional interference into system. This additional interference leads to increase in BER. To reduce the signal distortion means (low BER), we need to work in linear amplifier region with a large acceptable range. However, this linear amplifier has imperfect efficiency and also be expensive.

B. Distortions

Whenever the OFDM signal is clipped, it will leads to introduction of in-band distortion and out of band radiation into the wireless communication system. Thus the best solution is to reduce PAPR before OFDM signals enters the region of nonlinear HPA and DAC.

C. Power savings

Whenever HPA leads to high dynamic range, it means it exhibits worst efficiency. It shows that PAPR reduction cansignificantly saves the power [6]. Hence as the battery will reduced, the size of handset will be reduced to give the same backup.

D. Increased data coverage

Power effectiveness is very requisite in wireless communication as it provides adequate data coverage.

V. REDUCTION CRITERIA OF PAPR IN OFDM

A. BER degradation

The PAPR reduction techniques should being such a way that the BER performance should not be degraded or least affected

B. Availability of Bandwidth

Bandwidth is the main resource of any wireless communication. It should not be increased because either we have to pay a large amount of money to use it or it is not available. So in that case we have to use such techniques like channel coding etc. And whenever the channel coding is used the loss in data rate is increased due to side information.

C. In band Distortion

In-band distortions that is due to rotation, attenuation, and offset on the received signal [7]. The reduction in PAPR should be made in such fashion that in-band radiation should be kept at the minimum level.

D. Out of Band Radiation

HPA non-linear characteristic, excited by a large input, causes out-of-band radiation that affects signals in adjacent bands [14]. Out of band radiation should be avoided while reducing PAPR because it will affect nearby systems.

E. Low Average Power Concept

For reduction of PAPR it requires large linear operation in HPA, but it will increase the cost. The low average power can increase the BER in a system.

F. Implementation Complexity

If we have to achieve a good PAPR reduction at the cost of high complexity, then in that case hardware cost will be more and it can have lots of delay and consume a lot of time in processing. FFT techniques to implement the modulation and demodulation functions increase the computational efficiency of OFDM system [8].

VI. DISTINCT PAPR REDUCTION TECHNIQUES IN OFDM SYSTEM

In this section, we mainly discuss two typical techniques for PAPR reduction in OFDM systems and later we discuss PAPR reduction for multi users in OFDM.

A. Clipping and Filtering

It is one of the simplest techniques for PAPR reduction i.e. amplitude clipping. Amplitude clipping limits the peak envelope of the input signal to a predefined value,

$$B(x) = \begin{cases} x, & |x| \leq A \\ Ae^{j\theta(x)}, & |x| > A \end{cases} \quad (8)$$

Where $\theta(x)$ is the phase of x and A is the clipping threshold. The noise generated by amplitude clipping falls both in-band and out-of-band. In-band distortion cannot be reduced by filtering and results in error performance degradation, while out-of-band radiation reduces the spectral efficiency. After clipping, filtering the out-of-band distortion causes some peak regrowth and to avoid this undesirable effect, repeated clipping and filtering can be used [3][4][5]. Through the clipping technique OFDM signal without PAPR reduction is 4.0965 and after PAPR reduction is 2.9519.

OFDM signal without PAPR reduction is 13.4376 and after PAPR reduction through clipping and filtering is 10.9113.

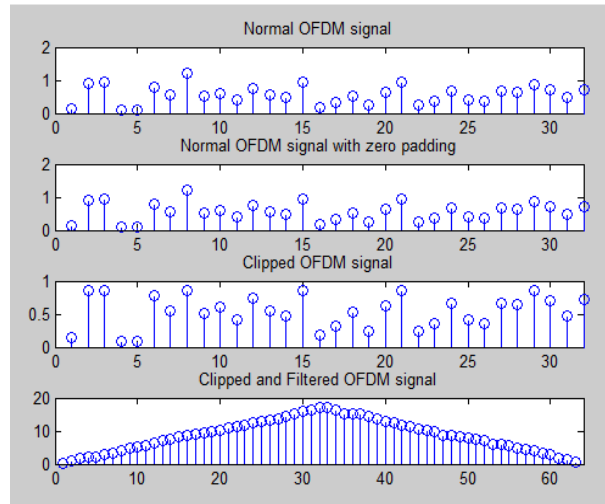


Fig.2. OFDM signal without PAPR reduction and with PAPR reduction by clipping and filtering technique.

B. Selected Mapping (SLM)

In the SLM technique, the transmitter generates a set of data blocks, where all representing the same information which the original data block contains, and we have to select the most favourable for transmission [9][10]. A block diagram of the SLM technique is shown in fig.3.

Each data block is multiplied by U different phase sequences, and each of length N ,

$$B^{(u)} = [b_{u,0}, b_{u,1}, \dots, b_{u,N-1}]^T, \quad u = 1, 2, 3, \dots, U,$$

resulting in U modified data blocks, we set $B^{(1)}$ as the all-one vector of length N .

Suppose the modified data block for the u_{th} phase sequence

$$D^{(u)} = [b_{u,0}, D_1 b_{u,1}, \dots, D_{N-1} b_{u,N-1}]^T \quad u = 1, 2, \dots, U$$

After applying SLM to X , the multi-carrier signal becomes

$$d_k^{(u)} = \sum_{n=0}^{N-1} D_n \cdot b_{u,n} \cdot e^{j \frac{2\pi nk}{N}} \quad 0 \leq k \leq N-1. \quad (9)$$

From the modified data blocks, we have to select the one with lowest PAPR for the transmission of data blocks. On the receiver side, the reverse operation is performed to recover the original data block. For doing implementation, the SLM technique needs U times Inverse DFT operations, and the number of required side information bits is for each data block.

This approach is applicable for all types of modulation and any number of subcarriers. The amount of PAPR reduction in SLM technique will depend on number of phase sequences U and the design of phase sequences.

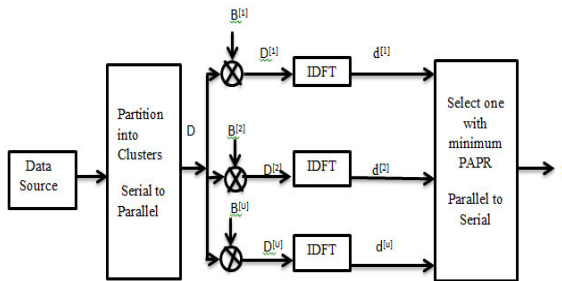


Fig.3. Block diagram of SLM technique

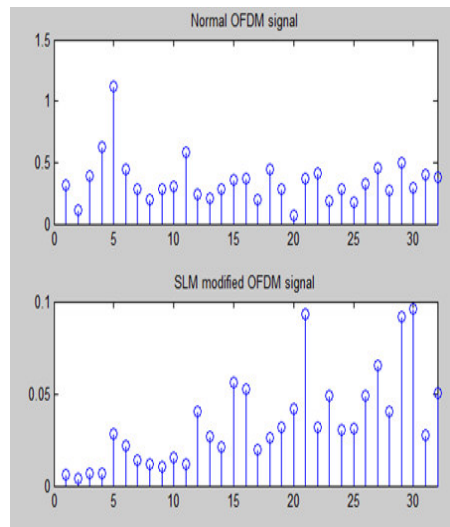


Fig.4. OFDM Signal without PAPR reduction, OFDM Signal after PAPR reduction.

In SLM technique, PAPR of normal OFDM is 15.0115 and PAPR of SLM modified OFDM is 11.9887.

C. Constant Modulus Algorithm(CMA) in OFDM/A MIMO

It is a new peak-to-average power ratio (PAPR) reduction approach for MIMO-OFDM/A is

developed based on well-known constant modulus algorithm (CMA). This approach has two ideas: 1) time domain signals from "resource blocks" (consisting of several subcarriers) may be linearly combined using precoding weights, which is transparent to the receiver; 2) these weights can be designed in such a way to minimize the modulus variations of developed signal, leading generally to a reduction in PAPR. This technique is suited with various beam forming modes in single antenna and MIMO systems.

Here we consider a generic MIMO-OFDM/A downlink scenario having one base station (BS) employing M_t antennas. An OFDM block with N subcarriers is transmitted from each antenna. The N subcarriers include N_u useful subcarriers surrounded by two guard bands with zero energy. These useful subcarriers are further grouped into M resource blocks (RBs) each consisting of

$$N_b = N_u / M \text{ subcarriers.}$$

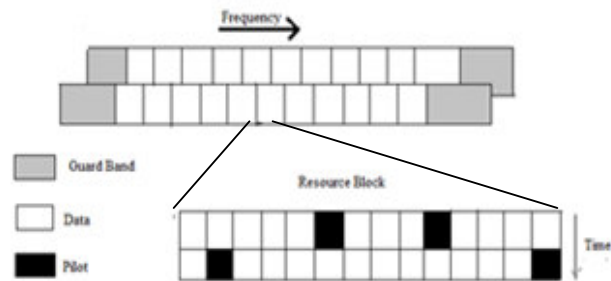


Fig.5. MIMO OFDM/A downlink data structure of an OFDM block.

$$\mathbf{X} = \begin{bmatrix} \text{CB} & \text{RB}^{(1)} & \text{RB}^{(2)} & \dots & \text{RB}^{(M)} & \text{CB} \end{bmatrix} \begin{bmatrix} \mathbf{W}^{(1)} & \mathbf{W}^{(2)} & \dots & \mathbf{W}^{(M)} \end{bmatrix}$$

Fig.6. Beam formed MIMO transmit data in frequency domain.

In MIMO transmit model we consider only a single time block. The data in the q -th RB is a matrix $\mathbf{e}^{(q)}$, which is pre-multiplied by a corresponding beamforming matrix $\mathbf{W}^{(q)}$, $q=1, \dots, M$, resulting in transmit sequences $\mathbf{x}^{(q)}$. The data model is

$$\mathbf{X} = \mathbf{W}^H \mathbf{D} \quad (10)$$

Where $\mathbf{W} = [\mathbf{W}^{(1)H}, \dots, \mathbf{W}^{(M)H}]^H$, and \mathbf{D} is a block-diagonal matrix structure as in Fig.6, which includes guard intervals as well. Here matrix \mathbf{X} , represents the spatial data the frequency domain.

In MIMO-OFDM transmit data model is obtained by taking IDFT of \mathbf{X} which is beamformed data matrix, resulting in

$$\mathbf{Y} = \mathbf{X} \mathbf{F}^H = \mathbf{W}^H \mathbf{D} \mathbf{F}^H \quad (11)$$

Where $\mathbf{F}^H \in \mathbb{C}^{N \times N}$ denotes IDFT matrix and $\mathbf{Y} \in \mathbb{C}^{M_t \times N}$ contains resulting transmit OFDM sequences for each M_t antennas.

Let time-domain data matrix i.e. $\mathbf{B} = \mathbf{D} \mathbf{F}^H$; this is a full matrix. Beamformed OFDM block can be expressed as

$$\mathbf{Y} = \mathbf{W}^H \mathbf{B} \quad (12)$$

Total Power in data matrix \mathbf{D} is $P_d = \|\mathbf{D}\|_F^2 = \|\text{vec}(\mathbf{D})\|_2^2 = \alpha N_t$ where $N_t = NM_t$. Function $\text{vec}(\mathbf{D})$ creates a column vector whose elements are the columns of matrix \mathbf{D} .

PAPR is a common term to measure the distortion caused by probable high peak of the OFDM signal and for a MIMO-OFDM block \mathbf{Y} we define,

$$\text{PAPR}(\mathbf{Y}) = \frac{\alpha N_t \|\text{vec}(\mathbf{Y})\|_\infty^2}{\|\text{vec}(\mathbf{Y})\|_2^2} \quad (13)$$

In this technique, lowest PAPR is achieved, for which infinity norm is equal to the average power of the sequence. MIMO-OFDM transmit matrix (replacing \mathbf{Y}) is

$$\mathbf{S} = \mathbf{W}^H \mathbf{\Omega} \mathbf{D} \mathbf{F}^H \quad (14)$$

Here, we define $\omega = \text{vec}(\mathbf{\Omega})$, then PAPR reduction problem is to design ω as :

$$\text{Min } \omega \|\text{vec}(\mathbf{S})\|_\infty^2 \text{ s.t. } \|\text{vec}(\mathbf{S})\|_2^2 = P \quad (15)$$

where $P = \alpha N_t$ is a fixed total transmit power.

(1) Approach of CMA

By Using the properties of Kronecker Products, we can rewrite equation (14) for \mathbf{S} as

$$\mathbf{S} = \text{vec}(\mathbf{S}) = (\bar{\mathbf{B}} \mathbf{W})^H \text{vec}(\mathbf{\Omega}) =: \mathbf{A} \omega \quad (16)$$

Where $\mathbf{A} \in \mathbb{C}^{M_t \times NM_t}$, $\mathbf{D} \mathbf{F}^H = \mathbf{B} \in \mathbb{C}^{M_t \times N}$, $\bar{\mathbf{B}}$ denotes the complex conjugate of \mathbf{B} , here denotes Khatri-Rao Product (column wise Kronecker Product).

Here we are using Unit-circle CMA (UC-CMA). In this technique we are having an i -th iteration i.e.

$$\hat{\mathbf{S}}^i = \mathbf{A} \mathbf{w}^i \quad (17)$$

$$\mathbf{w}^{i+1} = \mathbf{w}^i - \mu \nabla J(\mathbf{w}^i) = \mathbf{w}^i - \mu \mathbf{A}^T \hat{\mathbf{S}}_e^i \quad (18)$$

In order to restrict the solution to be on the unit circle, a normalization step is added to each iteration after (18)

$$\mathbf{w}^{i+1} \oslash |\mathbf{w}^{i+1}| = \mathbf{w}^{i+1} \quad (19)$$

Where \oslash is pointwise division and $|\cdot|$ takes the absolute value of each entry of vector argument. This is the alternative updating algorithm which is called as Unit circle CMA (UC-CMA).

VII. RESULT

For PAPR reduction, first we go through SISO (Single Input Single Output) i.e., for single antenna here we notice, that before PAPR reduction the signal is 8.081486 and after PAPR reduction, the signal is 5.057917.

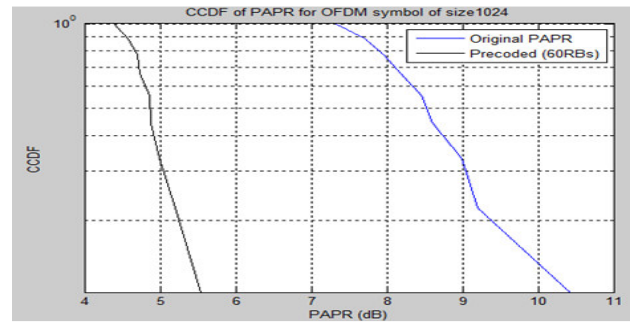


Fig.7. PAPR reduction performance in MIMO-OFDM with original PAPR and after Precoded

Here, the CCDF Curve shows the superior performance as compare to PAPR reduction in single antenna. The above curve shows the performance of UC-CMA for various number of transmit antennas, $M_t = 1, 2, 4,$ and 10 iteration.

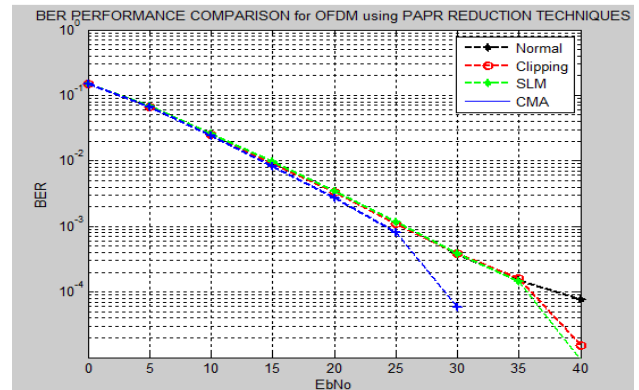


Fig.8. Comparison of BER based on different PAPR reduction.

An efficient PAPR reduction is that in which we achieve the lowest possible value of PAPR while keeping a minimal level BER.

TABLE 1.COMPARISON OF DIFFERENT PAPR REDUCTION TECHNIQUES

Techniques	Complexity	PAPR Reduction	BER = 4dB @ Eb/N0=	Power consumption	Bandwidth Expansion	Application
Clipping	Low	1.1446	38dB	Low	No	Short Distance, Low Speed
SLM	Moderate	3.0228	35.5dB	High	Yes	Long Distance, Low Speed
Clipping and Filtering	Low	2.5263	37dB	Low	No	Indoor, Moderate Speed
CMA	High	3.0235	27dB	Low	Yes	Indoor, Ultra-wide band, High Speed

VIII. CONCLUSION

Due to spectrum efficiency and channel robustness, OFDM is a very attractive technique for wireless. The major drawback in OFDM is its high PAPR when input sequences are highly correlated. Here we described mathematical analysis including average or envelope power in OFDM systems. Three typical techniques are there in this paper to reduce PAPR which have been analyzed, and all of which have potential to provide substantial reduction in PAPR at the cost of loss in data rate, transmit signal power increases, BER performance degradation, complexity increases. Here we also reduce PAPR for multiuser OFDM systems.

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