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Denoising Gaussian Noise Using Wavelet Packets

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Abstract: Noise is often introduced in an image during its acquisition or transmission. This noise can be present in one or more forms including the impulsive noise, the additive noise and the multiplicative noise. The effect of this noise is a decrease in the performance of visual and computational analysis. Hence, denoising of images corrupted with such noises is of vital importance for true analysis of the image. A variety of statistical and transform techniques have been proposed in the literature each with their own strengths and limitations. This paper proposes an adaptive wavelet packet based filter for denoising images contaminated with Gaussian noise. Five wavelets were meticulously chosen after an extensive study of various wavelets on a number of benchmark images. The noisy image was decomposed with each of these wavelets using wavelet packets transforms to obtain optimal wavelet base. Using the modified Bayes thresholding, and applying inverse wavelet packet transforms, the denoised image was obtained for each of these five wavelets. The median of every pixel is found from these outputs to obtain the final denoised image. The output obtained through this technique is found superior to various methods both qualitatively and quantitatively.

Keywords: Denoising, Wavelets, Wavelet Packets, Bayes Thresholding, Median, Gaussian Noise

I. INTRODUCTION

Digital images[1],[2] are susceptible to a number of factors which induce noise[1],[2] in them during the process of acquisition or transmission. These factors include sensor imperfections, transmission channel defects and physical constraints. A wide variety of noise exist in the image processing system. The prominent among them are the impulse noise, additive noise and multiplicative noise. Impulse noise is usually characterized by some portion of image pixels that are corrupted, leaving the remaining pixels unaltered. In images corrupted by additive noise, a specific value from a certain distribution is added to each image pixel, whereas for images corrupted with multiplicative noise, a particular value from a certain distribution is multiplied with the intensity of each pixel.

Digital images can be conveniently represented and manipulated as matrices containing the light intensity or color information at each spatially sampled point. The term monochrome digital image, or simply digital image, refers to a two-dimensional light intensity function $f(i,j)$, where i and j denote spatial coordinates, the value of $f(i,j)$is proportional to the brightness (or gray level) of the image at that point, and i, j and $f(i,j)$are integers. The problem of image denoising is to recover an image $f(i,j)$, from the observation $X(i,j)$, which is distorted by noise (or noiselike degradation) $n(i,j)$. The noisy image $X(i,j)$ can be segregated into a desired component, $f(i,j)$ and the noise component, $n(i,j)$. The most common decomposition is additive as given in equation (1).

$$X(i,j) = f(i,j) + n(i,j) \tag{1}$$

For instance, Gaussian noise[1],[2] is usually considered to be an additive component. The additive model is most appropriate when the noise is independent of $X(i,j)$. There are many applications of the additive model. Thermal noise, photographic noise, and quantization noise, for instance, obey the additive model well. Image addition also finds applications in image morphing.

The goal of any denoisingtechnique is to eliminate or reduce the noise while retaining as much as possible the important signal characteristics such as the edges and sharp features. In general, there are two fundamental approaches to image denoising viz.spatial filtering method and transform domain filtering method. The former approach can be applied for reduction of all the three types of noise while the latter can be applied for the additive and multiplicative noise reduction. The spatial method is preferred in the case of salt-and-pepper noise reduction while wavelets and their variants have become important for reduction of the other two types of noises.

The spatial[3]-[6] domain techniques directly deal with the pixels of the image. The pixel values are manipulated to achieve the desired output. As the effect of these techniques effect the whole image in a uniform manner, it is not possible to selectively deal with the edges and details effectively. This often results in undesirable outputs. On the other hand, the Transform coding techniques use a reversible, linear mathematical transform to map the pixel values onto a set of coefficients that are then quantized and encoded. The transform-based coding schemes employ the ability of the integral mathematical transforms to encode a significant amount of the visual information originally present in the image using a few coefficients. Consequently, most of the resulting coefficients have small magnitudes and can be quantized (or discarded altogether) without causing significant distortion in the decoded image. Important transforms include the Discrete Cosine Transforms, Fourier Transforms and the Wavelet transforms[7]-[11]. The Fourier transform, though simple and efficient, suffers from the drawback that part of image having frequencies similar to noise are also removed along with noise. This can cause significant loss of information of the desired image. Thus, conserving parts of the image having high frequencies similar to that of noise, was, to a great extent, overcome by the advent of wavelets.

Unlike the Fourier transform, wavelets can have infinite varieties which are fundamentally different from each other. The ones which have strictly finite extent in the time domain, are known as discrete wavelets, otherwise they are known as continuous wavelets. The Wavelet coding techniques are also based on the idea of using a transform that decorrelates the pixels of the input image, converting them into a set of coefficients that can be coded more efficiently than the original pixel values themselves. The wavelet analysis consists of decomposing a signal or an image into a hierarchical set of approximations and detail. The levels in the hierarchy often correspond to those in a dyadic scale.

In the orthogonal wavelet decomposition method, the basic step is to split the approximation coefficients into two parts. This produces a vector of approximation coefficients and a vector of detail coefficients. The information lost between two successive approximations is captured in the detail coefficients. This is again followed by splitting the new approximation coefficient vector again into two parts. However, the successive details are never split. On the other hand, in the wavelet packets[1],[12], each detail coefficient vector is also decomposed into two parts using the same approach as in approximation vector splitting. This offers a very rich analysis by producing a complete binary tree. Thus, Wavelet packet transform is a generalization of wavelet transforms that offers a rich set of decomposition structures.

This paper proposes a novel technique that employs adaptive wavelet packet transform based filter to denoise images corrupted with Gaussian noise.

The rest of the paper is arranged as follows. The proposed methodology is introduced in section II while simulation results are given in section III. The section IV includes the conclusion.

II. PROPOSED METHODOLOGY

This technique incorporates adaptivity into the multiple Wavelet Packets for enhancing its performance. Wavelet-packet transforms offer a rich set of decomposition structures. Wavelet packet transformation is achieved when the filter bank is iterated over all frequency bands at each level. Wavelet packets have better ability to represent high-frequency content and high-frequency oscillating signals. This allows a wavelet packet transform to perform better than wavelets for denoising.

The adaptive wavelet packets use optimal wavelet packet bases for decomposition. In this type of transform, the optimal representation basis of the input signal is selected by optimization of a “cost function” in each subband. This method selects a level and subband dependent adaptive threshold value on the basis of the statistical parameters of subband coefficients. The process of obtaining the Optimal Wavelet Base(OWB)extraction[13]-[14] and determining the threshold[15]-[21] value are now discussed.

A) OWB extraction

For extracting the optimal basis from the full WP tree of an input image, the conventional algorithm uses the bottom-up approach. The algorithm begins at the deepest level of the tree and removes quads of nodes that have cost higher than the parent node at each level while working backwards toward the root.

In order to reduce computational complexity of this algorithm, an alternative fast method for extracting OWB, has been proposed in [13]. This is a top-down search algorithm that starts at the root and generates the optimal basis tree without growing the tree to full depth. Use of Shannon entropy is made in this algorithm to produce the optimal wavelet basis. The pseudo code of this algorithm is shown below:

Algorithm of Fast OWB Extraction[13]**:**

Step 1: Select the highest number of levels, L, for Wavelet packet decomposition.

Step 2: Determine the optimal basis for Wavelet Packet tree. Till the current level (d) of decomposition is less than L, the following steps are carried out for each existing subband (parent node) S_d^i $(0 \leq i < 4^d - 1)$.

i) Computation of the subband’s Shannon entropy $SE(S_d^i)$as the cost function.

ii) Decomposition of S_d^i into four subbands (children nodes LL_{d+1}^{4i}, LL_{d+1}^{4i+1}, LL_{d+1}^{4i+2} and LL_{d+1}^{4i+3}) and computation of the Shannon entropy for each of them: $SE(HL_{d+1}^{4i})$, $SE(HL_{d+1}^{4i+1})$,$SE(HL_{d+1}^{4i+2})$ and $SE(HL_{d+1}^{4i+3})$.

iii) If $SE S_d^i < SE(HL_{d+1}^{4i}) + SE(HL_{d+1}^{4i+1}) + SE(HL_{d+1}^{4i+2})$ and $SE(HL_{d+1}^{4i+3})$, then only the parent node is retained while children nodes are eliminated; else both parent and children nodes are retained.

iv) When there are no nodes to split, the process of extracting the OWB is terminated

B) Threshold value determination

To determine an optimal value for thresholding is not easy. It the threshold is set to a large value, then a larger number of coefficients are made equal to zero leading to smoothing of the image. This may result in producing blurs and artifacts, finally leading to loss of useful information from the signal. On the other hand, if the threshold is set to a small value, then it may allow noisy coefficients in the output image. Hence, it is necessary to choose an optimum threshold value that varies as per the subband characteristics so as to bring out the image with minimum amount of noise. An optimum threshold selection algorithm using Bayes Shrink has been used. In this algorithm proposed in [14], an adaptive threshold value $\lambda_s$for each subband$S$ at level d is calculated as

$$\lambda_s = \alpha_{d,s} \frac{\sigma_\eta^2}{\sigma_{f,s}} \tag{2}$$

where σ_η^2 and $\sigma_{f,s}^2$ are the variances of noise and the noise-free image coefficients in the subband$S$, respectively. The term $\alpha_{d,s}$helps in making the threshold adaptive so as to make it suitable for different decomposition levels and subbands i.e. $\alpha_{d,s}$is set for a larger threshold values for high-frequency subbands based on their level of decomposition and their corresponding subbands while it is set for smaller threshold values for low-frequency subbands. Thus $\alpha_{d,s}$makes the threshold value dependent on the decomposition levels and each of the subbands. As more information in the image is present in the low-frequency subbandcompared in the high-frequency subband and as the chances of presence of noise in the high-frequency component is greater, applying a greater threshold value to the high-frequency subbands decreases the effect of noise more effectually.

A subband weighting function (SWF) in horizontal (SWF_H)and vertical (SWF_V)directions at level L of the Wavelet Packet decomposition was proposed in [14]. This function given by equation (3) is an increasing function in both directions.

$$SWF(i) = \frac{i^2}{2^{2L}} \text{for } i= 1, 2, \ldots, 2L \tag{3}$$

where$i$is the index of subbands at the highest level of decomposition in horizontal and vertical directions. The value of $\alpha_{d,s}$for each subbandS at each level d is calculated as the sum of SWF values in horizontal (SWF_H)and vertical (SWF_V) directions that are spanned by subband Sas shown in eq. 4

$$\alpha_{d,s} = \sum_{i \subset s} SWF_H(i) + \sum_{j \subset s} SWF_V(j) \tag{4}$$

Thus, the threshold value calculation is level and subband dependent.

The procedure for reducing Gaussian noise using the Adaptive Wavelet Packet based thresholding using BayesShrink is described as follows:

Stage 1: Decomposition of the corrupted image with db8 wavelet at level N by Wavelet Packets decomposition to obtain Optimal Wavelet Base of a noisy image using Shannon's entropy.

Stage 2: Estimation of the noise variance.

Stage 3: Computation of the threshold value and the statistical parameters for each subband S in level d as follows[20]:

a) Determination of the subband's variance using eq. 5.

$$\sigma_x^2 = \frac{1}{n^2} \sum_{i,j=1}^{n} X^2(i,j) \tag{5}$$

b) Pre-estimation of the variance of original image using eq. 6.

$$\sigma_f^2 = \max\left(\sigma_x^2 - \sigma_n^2,\ 0\right) \tag{6}$$

c) Calculation of the term $\alpha_{d,s}$ using eq. 4.

d) Computation of the thresholding value using eq. 2.

Stage 4: Thresholding of all subband's coefficients using the soft thresholding technique.

Stage 5: Application of the inverse WPT to reconstruct the denoised image, y_1.

Stage 6: Repetition of the last five steps using the remaining four wavelets to obtain the outputs $Y_2(i,j)$, $Y_3(i,j)$, $Y_4(i,j)$and $Y_5(i,j)$

Stage 7: Calculation of the median for each pixel location using the corresponding pixel values of the five outputs to obtain the final denoised output, Y(i,*j*).

III. SIMULATION RESULTS

Quantitative and qualitative evaluation in the form of PSNR and visual inspection were carried out on the filtered images obtained by this technique for judging the effectiveness of the filters in removing Gaussian noise. A wide range of noise levels varying from σ = 5 to 50 in steps of 5 have been used for performance assessment and the results tabulated as shown below.

TABLE 1
Comparison of Restoration Results of Multiple Adaptive Wavelet Packets Filter (MAWPF) for 'Lena' Image in terms of PSNR(dB)

Noise (σ)→ / Filtering technique↓	5	10	15	20	25	30	35	40	45	50
INPUT PSNR	34.16	28.15	24.60	22.12	20.22	18.69	17.44	16.34	15.42	14.61
WIENER FILTER[1],[2]	37.10	34.07	31.36	29.15	27.35	25.88	24.63	23.58	22.69	21.90
AVERAGE FILTER[4]	35.9	31.3	28.1	25.5	23.64	22.5	21.43	20.2	19.73	18.4
VISUSHRINK[15]	34.3	28.2	24.6	22.1	20.67	18.7	17.34	16.4	15.73	14.6
SURESHRINK[17]	25.1	25.1	25.1	25.1	25.1	25.0	24.9	24.8	24.7	24.6
WP FILTER USING BAYES THRESHOLDING	37.87	33.95	31.92	30.57	29.61	28.89	28.06	27.46	26.95	26.37
ADAPTIVE WP FILTER USING BAYES THRESHOLDING	38.02	34.07	32.05	30.63	29.67	28.78	28.12	27.48	26.99	26.26
PROPOSED METHOD (MAWPF)	38.35	34.56	32.51	31.10	30.02	29.24	28.48	27.87	27.29	26.87

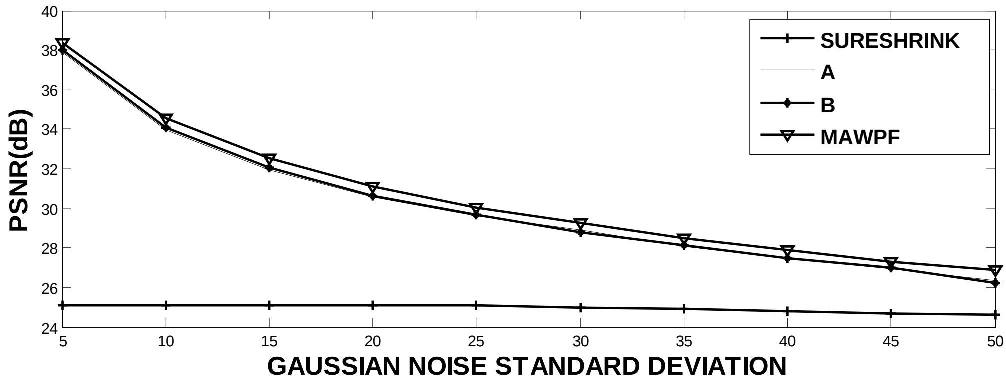
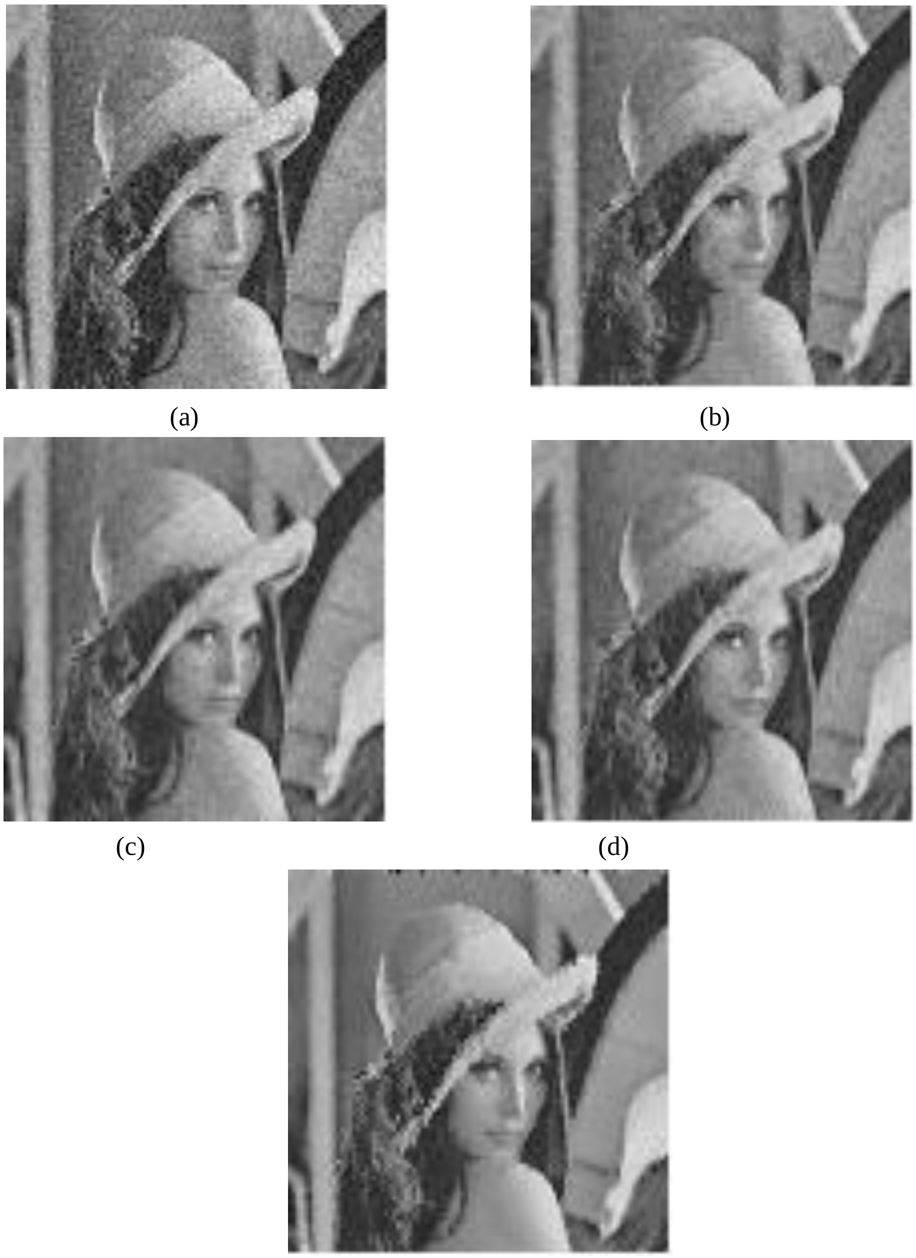


Fig. 1. Restoration Results for 'Lena' Image. Here A represents the WP filter using Bayes Thresholding, while B represents Adaptive Wavlet Packet Filter using Bayes Thresholding

Table 1 lists the restoration result in PSNR (dB) of the MAWPF for 512 x 512 grayscale image 'Lena' corrupted by Gaussian noise of various noise levels. Fig. 1 shows the result graphically while qualitative analysis can be seen in Fig. 2.



(a) (b) (c) (d) (e)

Fig. 2. Restoration Results for σ=50 for Various Methods (a) Visushrink (b) Sureshrink (c) A (d) B (e) MAWPF

It can be seen that this hybrid algorithm performs significantly better than the VisuShrink and SureShrink based filters at both low and high noise levels. This can be seen both at the quantitative and qualitative levels. Thus, the Adaptive Wavelet Packet based thresholding using BayesShrink filter proposed in this section can reduce the Gaussian noise more efficiently while preserving the edges at both low and high noise levels as compared to other state-of-the-art methods.

IV. CONCLUSION

This paper proposes an adaptive Wavelet Packet based filter for denoising images contaminated with Gaussian noise. Wavelet Packets offers a rich set of decomposition structures as they decompose both the approximation and detail making them exhibit better ability to represent high-frequency content and high-frequency oscillating signals. Five wavelets are meticulously selected which is followed by using each of them to decompose the corrupted image at various levels using Wavelet packets approach to obtain optimal wavelet base using Shannon's entropy. This is followed by calculation of the median for each pixel location using the corresponding pixel values of the five outputs to obtain the final denoised output. The result of this technique outperforms many other methods for denoising images corrupted with Gaussian noise of various levels. However, it should be noted that using wavelet packets, the processing time is increased. Future work may be in the direction of reducing this processing time.

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