

# Queuing Theory Applications: A Review

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**Abstract:** The main goal of queuing system is to reduce the waiting time and provide quick service. Many different performance measures can be derived and calculated with the help of this theory. To improve the efficiency, a proper understanding of physical environment controlling of formation of queue is required. Queuing theory is branch of operation research. Queuing Theory applications are used for making decision about resources needed to provide service. Real life models of queuing theory cover a wide range of applications, such as how to provide faster customer service, improve traffic flow design of telecommunication system, call center, banking sector, Hospital, Toll Plaza, Malls, Restaurants, Marriage party dinner, Lunch etc. This paper reviews the basic features of queuing theory and its applications to reduce the waiting time.

**Keywords:** Queuing System, Queuing Theory, Queue length, Waiting time, Arrival process, Service process.

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## Introduction

Queues happen when resources are limited. Queuing theory helps in design in balance system quickly and effective at most elementary level. It involves the analysis of arrivals at facility such as traffic flow, banks, post office, malls, fast food restaurants, programme on computers, jobs on call centers etc. The father of the queuing theory is Agner Krarup Erlang (1878-1929) a Danish engineer, statistician and mathematician [6]. He published the article study of congestion in field of telephone network analysis. Queuing theory, an operation management technique is commonly used to determine flow of units for service, forming or joining the queue and customer service shortfall.

**Parameters of Queuing System:** Parameters of Queuing System are as :

### Queue length $L(t)$ :

Queue Length refers to number of customers waiting in queue at any time. With Probability Distribution of arrival and service process, the Probability Distribution of Queue length can be obtained.

### Mean Queue Length:

The mean queue length is the average number of units waiting in queue for being serviced .

### Idle Period:

Duration of time during which customers are not served is known as Idle Period. Server is idle during this period.

### Busy Period:

Server is busy and is serving the customer during this busy period.

### Waiting Period:

If the customer joins the queue at time  $t$ , he has to wait in system for completion of his service during this time. If  $W_q$  is the average waiting time in the queue,  $W$  is the average time in the system and  $W_s$  is the average service time.

$$\text{Then, } W = W_q + W_s$$

For a steady state queuing system, Lee (1961) established the following formula

$$L = \lambda W \quad \text{and} \quad L_q = \lambda W_q$$

Where  $\lambda$  is the mean arrival rate.[7]

**Characteristics of Queuing System:** Characteristics of Queuing system are as follows:

**Arrival Pattern of Units (Input Source):**

The method in which units arrive and join the queue system is called as the input. This is considered as the main characteristics which shows size i.e. population. That is the total no. of customers which can be finite or infinite, who want to be serviced.

The process of arrivals is stochastic. It may be regular arrival, independent arrival, completely random arrival, discrete time arrival, aggregated arrival, Markovian arrival and Non Markovian arrival. Arrivals depend upon the number of customers waiting for service according to the customer's behaviour as the customers are sometimes attracted or detracted by long queue.

**Order Pattern of service (Queue):**

Maximum permissible customers who want services can be finite or infinite. The order in which units are selected from a queue for a service out of those waiting represents the queue discipline. Some disciplines are first come first served (FCFS), last come first serve (LCFS) and service in random order (SIRO). The most common discipline is FCFS.

**Service Pattern of units (Service):**

In this, the service facilities contains one or more parallel channels. The service facility can be one or more than that in number. Customer service may happen individually or in batch. Some of the types of channel are as

1. Network of multiple Channel
2. Channel in Series
3. Channel in Parallel
4. Channel in Parallel and Series

**Customer Behaviour:**

Three types of customer behaviour is observed:[5]

**Balking:** In this customer behaviour, the customer doesn't join the queue as he finds the queue too long. Customer is said to have balked if he shows this kind of behaviour

**Reneged:** Here the customer loses patience in some time after entering the queue and decides to leave the queue.

**Jockey:** This kind of behaviour is seen when there are more than one queue. The customer can switch from one queue to another queue. This is done just to reduce the waiting time. This is commonly seen at Toll Plaza, Billing Counter of the Mall etc.

**Customer Source:**

Customer source is considered as the source from where the customers are generated for a queuing system. This is also referred as waiting space. The waiting space may be finite or infinite. If the system is fully occupied and the customer source is finite and arrival of a unit is there, then this unit is usually considered lost for the system.

**Kendall's description of a queuing system:**

The characteristics of queuing phenomenon can be summarized as[7]:

$$(A/B/C) : (D/E/F)$$

Where

- A refers to description of arrival distribution.
- B refers to service time distribution
- C refers to number of servers(1,2,3... ∞)
- D refers to Queue Discipline
- E refers to maximum number allowed in the system
- F refers to Capacity of the system. It may be finite or infinite.

Symbol A and B are generally of these forms:

M = Markovian arrival/departures distribution.  
D = Constant time distribution.  
G = General time distribution  
 $E_k$  = Gamma Distribution of time.  
GI = General distribution of inter-arrival time.

### Queuing Theory Process:

Various processes are used in Queuing Theory. Some important processes are as: Stochastic Process, Strictly Stationary Process, Renewal Process, Markov Process, Poisson Process, Birth and Death Process.

### Types of Solutions of Queuing System:

Basically two types of solutions are used:

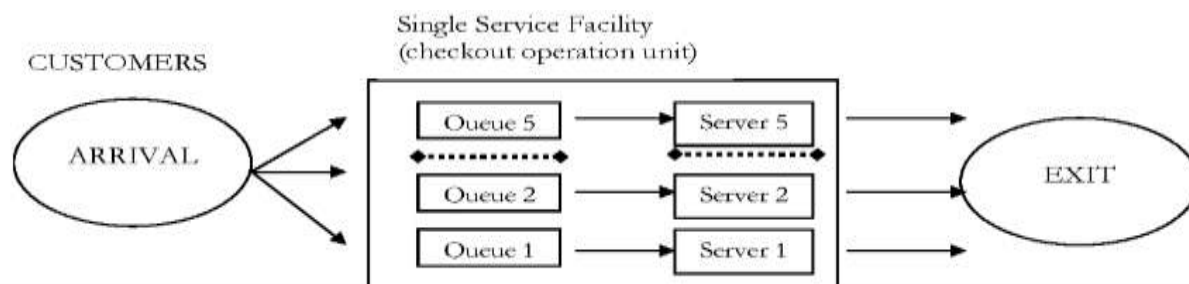
1. **Transit State Solution:** Transit Time solution is used when the arrival time, service time, waiting time and the operating characteristics of the customers are time dependent.
2. **Steady State Solution:** Steady State solution is used when the arrival time, service time, waiting time and the operating characteristics of the customers are time independent.[6]

**Queuing Systems:** Various types of queuing systems are follows:

1. Markovian's Queues:
  - a. (M/M/I) Model:
  - b. (M/M/S) Model:
2. Bulk Queues:
3. Correlated Queues
4. Serial and Parallel Channel
5. Non-Markovian Queues:
  - a. Phase Technique
  - b. Supplementary Variable Technique
  - c. Embedded Markov Chain Technique
  - d. Integral Equation Technique

**Applications of Queuing System:** In real life situations, queuing system can be used to reduce the waiting time by existing queuing models. Some of the situations are as follows:

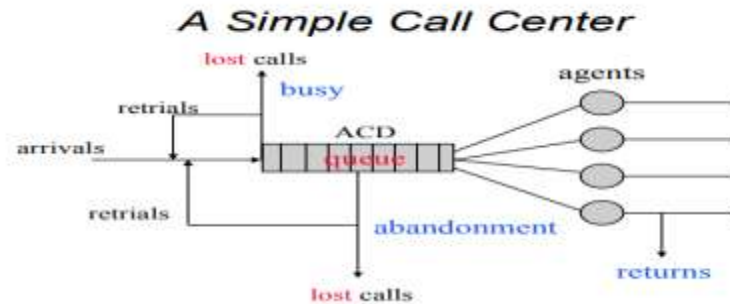
1. **Toll Plaza:** In toll plaza, M/G/I queuing model is used[3]. In this model design the toll collection methods, types of vehicles and number of toll booths configurations are used. Various performance parameters are being used. Some are as average waiting time, average queue length, maximum queue length and maximum waiting time. These performance parameters are combined with the designing factors such as number of manual tolls, electronic toll collection rates, lane selections. [4]



**Fig 1. Toll Plaza Model [1 ]**

To find the input parameters in appropriate manner, traffic simulation model is always considered as a challenge. Deterministic traffic counts for certain time can serve as better input parameter than probabilistic distribution parameters.

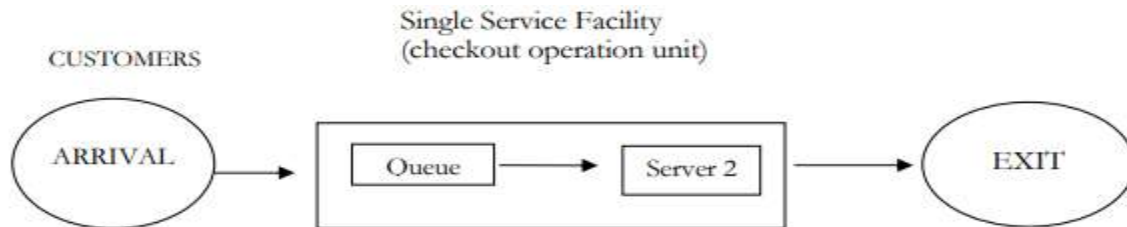
**2. Call Center:** In Call Center application, M/M/S queuing model is used[2]. Here the customers are referred to as callers, servers are telephone operators. The tele-queues are consists of callers that are in waiting state to be served by system resources.



**Fig. 2 Simple Call Center Scheme [ 2]**

This figure represents that there is single queue for the customers. Whenever the customer enters into the queue, depending on the availability of the system resource, it is served by any one of the available telephone operators.

**3. ATM:** In ATM, (M/M/I) queuing model is used. The technique used is Poisson Process. Here the service time and arrival time are exponentially distributed.[4] Here the single queue and single server is used in the model.



**Fig. 3 ATM Model [1]**

Here the service rate, arrival rate, waiting time and utilization rate in the queue is measured. The input parameters are the average number of customers in the queue.

**Proposed Application:**

Number of cases are pending in courts. People are waiting for their turn. People have lost confidence in Courts due to long delay in justice. If Queuing Theory will be applied with the suitable model by collecting primary and secondary data, can help to reduce the waiting period and enhance the judiciary process. A queuing theory model based on Poisson Probability Distribution can be designed to determine the parameters such as congestion, waiting period, delay. Some factors such as appropriate number of judges, establishment of new courts etc. that cause delay in the process should be solved so that the efficiency of the model can be increased.

**Conclusion:**

Queuing theory has many applications in real life. This theory plays important role in everyday life. Whenever the current demand for a service exceeds the current capacity, the formation of queue occurs.

Queuing Theory is used in many applications such as transportation networks, communication, hospitals, railway station, banks, traffic system, library management etc. This paper also provides a proposed queuing model with the existing queuing models.

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