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SIMULATION ASPECT OF AN ARTIFICIAL PACEMAKER

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The present work describes the design and analysis of a control system for regulating the heart rate using pacemaker in an efficient way. The total control system in this work is considered to be composed of cardio vascular system duly energized by one pacemaker system as operated in a closed loop manner with unity negative gain in the closed loop feedback path. The design emphasizes on the optimality in operation of the control process as determined by the performance index (PI) of the total process. For attaining the optimality in performance (as realized in terms of PI), one compensator is, however, used along with the cascade arrangement of the cardio vascular system driven by the concerned pacemaker. Hence, the overall open loop transfer function in the forward path is simply the product of the transfer functions of the pacemaker, the cardio vascular system and the compensator system. The controllability and the observability of the system are tested, and the system is found to be controllable & observable. MATLAB 7.1 software is suitably used in the concerned analysis and design.

Keywords: Performance Index, Pacemaker, Integral Square Error, Closeloop System, Optimal System, Controllability, Observability.

1. Introduction

One simplified model of a control system for regulating the heart rate of a patient in an efficient way is considered as shown in Fig.1.

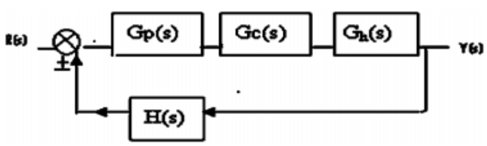

Fig.1: Block Diagram of Artificial Control of Heart Rate using Pacemaker

$G_p(s) = 8/(s + 8)$; $G_c(s) = K(s + 30)$; $G_h(s) = 169/(s^2 + 20s)$, H(s)=1; R(s) = actual heart rate; Y(s) = desired heart rate. Since the pacemaker system is basically one low pass system, allowing various excitations and rejecting some others, as evolved in normal metabolic process for the regular function of a cardiovascular system of a living being[4], the transfer function $G_P(s)$ of the pacemaker is suitably considered as that of a low pass filter with suitable cut off frequency. The transfer function $G_P(s)$ of the pacemaker is considered as given by the following expression: $G_P(s) = 8/(s + 8)$. Consideration is focused on the fact that the cardiovascular system is one under damped second order system having suitable parametric values of the damping factor and the natural frequency, so that the heart performs its normal function appropriately. Hence, the transfer function G_h (s) of the heart is chosen as given by the following expression [3]: $G_h(s) = 169/(s^2 + 20.8s)$. The compensator considered in the present design is one proportional plus derivative controller with its transfer function Gc(s) represented by the expression: G_c (s) = K(s+30). The design is so done that optimality in the performance index for the overall optimal control of the system is maintained. There are again various measures of PI. The concept of integral square error is, however, suitably considered in the present work. The present problem, hence, determines the optimal value of the gain(K) of the PD compensator.

The total control system is analyzed using MATLAB 7.1 software. The system designed in the present work is found to be stable with appropriate gain margin and phase margin. The controllability and observability of the total control system is tested. The system is found to be controllable and observable. The sample data analysis of the system is also done in the present study, and the system is also found to be stable in the sample data system, as done by Jury's test. Thus the control system designed in the present work is one sufficiently realizable system.

2. Conversion from s-Domain to z- Domain

The transformation of z transform helps in the analysis & design of sample data control system, as Laplace transform does in the analysis and design of continuous data control system.

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The z-transform F(z) of a sample data control signal f(KT) is defined by the relation:

$$F(z) = \sum_{K=0}^{\infty} f(KT) z^{-K} \quad (1)$$

The above relation is derived from the Laplace transformation as applied to sample data control signal, assuming $e^{sT} = z$ as the concerned transformation variable in Laplace transformation.

We have,

$$sT = \ln z \text{ i.e, } s^{-1} = \frac{T}{\ln z} \quad (2)$$

Using power series expansion of lnz, the above equation becomes:

$$s^{-1} = \frac{T}{2}\left[\frac{1}{u} - \frac{1}{3}u - \frac{4}{45}u^3 - \frac{44}{945}u^5\right] \quad (3),$$

where

$$u = \frac{1 - z^{-1}}{1 + z^{-1}} \quad (4).$$

In general, for any positive integral value n, we have,

$$s^{-n} = \left(\frac{T}{2}\right)^n \left[\frac{1}{u} - \frac{1}{3}u - \frac{4}{45}u^3 - \frac{44}{945}u^5\right]^n \quad (5)$$

By using binomial expansion in the above equation for various values of n, we may have the transformation from s to z domain.

3. PERFORMANCE INDEX AND OPTIMAL SYSTEM[2]

It is used to evaluate the system performance. A quantitative measure of the performance of a system is necessary for the operation of the modern adaptive control systems for automatic parameter optimization of a control system, & for the design of optimum systems." A performance index is a quantitative measure of the performance of the system and is chosen so that emphasis is given to the important system specification.

A system is considered a optimum control system when the system parameters are adjusted so that the index reaches an extremum value, commonly a minimum value.

A performance index, to be useful, must be a number that is always positive or zero. Then the best system is defined as the system that minimizes this index.

A system design problem generally reaches the point at which one or more parameters are to be selected to give the best performance. If a measure or index of performance can be expressed mathematically, the problem can be solved for the best choice of the adjustable parameter. The resulting system is termed optimal with respect to the selection criteria.

A commonly used performance index J is the integral of the square of the error to a step input. If the step error is expressed as a function of the adjustable parameters, the index can be minimized with respect to the parameter, yielding the optional parameter value. For the I_{SE} to be finite, the steady state value of the $e_{step}(t)$ must be of type 1,causing $e_{sstep}(\infty)$ to be 0. In the present study and analysis, e(t) and E(s) are assumed to be for a step input, the system is of type 1, and J refers to $I_{SE.}$

4. INTEGRAL SQUARE [2]

Instead of the time domain calculation of J (integral square error), the complex frequency domain can be used. According to a theorem in mathematics by parseval.

$$J = I_{SE} = \int_0^{\infty} e^2(t)dt = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} E(s)E(-s)ds \quad (6),$$

where E(s) can be expressed as follows:

$$E(s) = \frac{N_{n-1}s^{n-1} + \ldots + N_1 s + N_0}{D_n s^n + D_{n-1}s^{n-1} + \ldots + D_1 s + D_0} \quad (7)$$

asssuming type 1 behavior.

J follows from complex variable theory. To clarify the effect of system order, the subscript for J will be the system order. For an n^{th} order system, the performance index is given by the relation:

$$J_n = (-1)^{n-1}\frac{B_n}{2D_n H_n} \quad (8)$$

where H_n and B_n are determinants. H_n is the determinant of the n×n matrix. The first two rows of the Hurwitz matrix are formed from the coefficients of D(s), while the remaining rows consist of right-shifted versions of the first two rows until the n×n matrix is formed. Thus we write,

$$H = \begin{bmatrix} D_{n-1} & D_{n-3} & . & . & . \\ D_n & D_{n-2} & . & . & . \\ 0 & D_{n-1} & D_{n-3} & . & . \\ 0 & D_n & D_{n-2} & . & . \\ . & . & . & . & . \end{bmatrix} \quad (9)$$

The determinant B_n is found by first calculating

$$N(s)N(-s) = b_{2n-2}s^{2n-2} + \ldots + b_2 s^2 + b_0 \quad (10),$$

then first row of the Hurzwitz matrix is replaced by the coefficients of N(s)N(–s), while the remaining rows are unchanged.

$$B_n = \begin{bmatrix} b_{2n-2} & . & . & b_2 & b_0 \\ D_n & D_{n-2} & . & . & . \\ 0 & D_{n-1} & D_{n-3} & . & . \\ 0 & D_n & D_{n-2} & . & . \\ . & . & . & . & . \end{bmatrix} \quad (11)$$

5. OPTIMAL SYSTEM, CONTROLLABILITY & OBSERVABLITY

Optimal system is a system whose parameters are adjusted so that the performance index reaches an extremity value is considered as one optimal system.

5.1. Optimal Control: [13]

Optimal control is one particular branch of modern control that sets out to provide analytical designs of a specially appealing type.The system that is the end result of an optimal design is not supposed merely to be stable, have a certain bandwidth, or satisfy any one of the desirable constraints associated with classical control, but it is supposed to be the best possible system of a particular type – hence, the word optimal. If it is both optimal and possesses a number of the properties that classical control suggests are desirable, so much the better.

5.2. Controllability

A system described by the matrices (A, B) can be said to be controllable, if there exists an unconstrained control u that can transfer any initial state x(0) to any other described location x(t).

For the system, the state equation is represented as:

$$x' = Ax + Bu$$

We can determine whether the system is controllable by examining the algebraic condition:

$$\text{Rank}[B\ AB\ A^2B\ldots\ldots A^{n-1}B] = n.$$

For a single input, single output system, the controllability matrix P_c is described in terms of A & B.

$$P_c = [B\ AB\ A^2B\ldots\ldots A^{n-1}B],$$

which is an nxn matrix. Therefore if the determinant of P_c is nonzero, the system is controllable11].

5.3. Observability

Observability refers to the ability to estimate a state variable. A system is observable, if and only if, there exists a finite time t such that the initial state x(0) can be determined from the observation history y(t) given the control u(t)[12]. Consider the single input single output system represented by the state space equation:

$$x' = Ax + Bu, \text{ and } y = Cx,$$

C is a row vector & x is a column vector. This system is observable, when the determinant of Q is nonzero where Q = C CA ……CA^{n-1} which is a n × n matrix.

The concept of controllability and observability were introduced by kalman. They play an important role in the design of control systems in state space. In fact, the conditions of controllability and observability may govern the existence of a complete solution to the control system design problem. The solution to this problem may not exist if the system considered is not controllable. Although most physical systems are controllable and observable, corresponding mathematical models may not possesses the property of controllability and observability. Then it is necessary to know the conditions under which a system is controllable and observable.

6. SYSTEM DESIGN

The overall control system under study consists of one appropriate compensator in cascade with the pacemaker system and heart system in closed loop manner with unity negative feedback path. Under present situation, overall transfer function of the system is given by the relation:

$$\frac{Y(s)}{R(s)} = T(s) =$$

$$\frac{1352k(s+1)}{s^3 + 28.8s^2 + (166.4 + 1352k)s + 40560k} \quad (12)$$

Thus for the entire system the characterstic equation is

$$s^3 + 28.8s^2 + s(166.4 + 1352k) + 40560k = 0 \quad (13)$$

For this characteristics equation to make the design problem with stability, the Routh array is constructed as below:

S^3 1 166.4+1352k

S^2 28.8 40560k

S^1 166.4-56.33

S^0 40560k

For stability

$0 < k < 2.95$

Now

$$T_E(s) = \frac{1 - T(s)}{s}$$

$$= \frac{s^2 + 28.8s + 166.4}{s^3 + 28.8s^2 + s(166.4 + 1352k) + 40560k} \quad (14)$$

$N_2 = 1$, $N_1 = 28.8$, $N_0 = 166.4$ (15)

$D_3 = 1$, $D_2 = 28.8$ $D_1 = 166.4 + 1352K$, (16)

$D_0 = 40560K$ (17)

$$J_3 = \frac{B_3}{2D_3H_3} \quad (18)$$

$$H_3 = \begin{bmatrix} D_2 & D_0 & 0 \\ D_3 & D_1 & 0 \\ 0 & D_2 & D_{0\partial} \end{bmatrix} \quad (19)$$

$$N(s) = s^2 + 28.8s + 166.4 \quad (20)$$

$$N(-s) = s^2 - 28.8s + 166.4 \quad (21)$$

$$N(s)N(-s) = s^4 - 496.6\, s^2 + 27688.96 \quad (22)$$

$$B_3 = \begin{bmatrix} b_4 & b_2 & b_0 \\ D_3 & D_1 & 0 \\ 0 & D_2 & D_0 \end{bmatrix} \quad (23)$$

$$J_3 = \frac{B_3}{2D_3H_3}$$

$$J_3 = 0.002\frac{(68.77k^2 + 33.72k + 1)}{(-0.34k^2 + k)} \quad (24)$$

Now, for maximum or minimum

$$\frac{dJ_3}{dk} = 0 \cdot$$

Simplifying we get

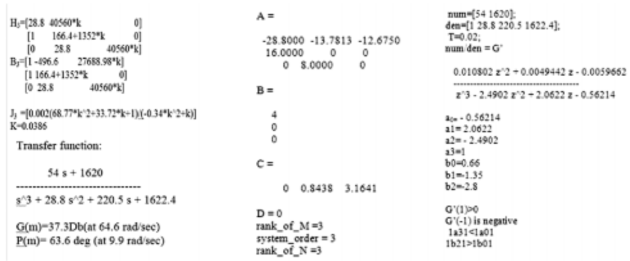
$$k = 0.0386 \approx 0.04, \quad (25)$$

is the only real & positive value. With this value of k, d^2J_3/dk^2 is positive, implying the availability of minimum extremum of the PI. Hence the design of optimality gets satisfied.

7. METHODS AND MATERIAL

Normally control system is considered in continuous data control system (continuous time domain ↔ Lapalace domain), the system analysis and study get restricted for any change in the system parameter, or input variation for easy and ready study. To circumvent this problem sample data(s.d.) control system makes study and analysis easy and ready available with variation in the system parameter and also the input. For this reason the system is also studied in sample data control model. The stability of the present system is tested by Jury's stability test which guarantees the stability of the overall system. Needless to mention, any stable system when operated in s.d. mode, the system is not necessarily to be guaranteed to remain stable in the s.d. mode also, there being the enhancement of the order of the system as introduced by the sampler. As any control system deserves to reach its steady state by which the system finally runs, and follows the input at that state, the designed parameter K is accordingly decided for minimum value of the error, the other desirable characteristic performances being also available in the system.

7.1. Matlab Output Files[6]

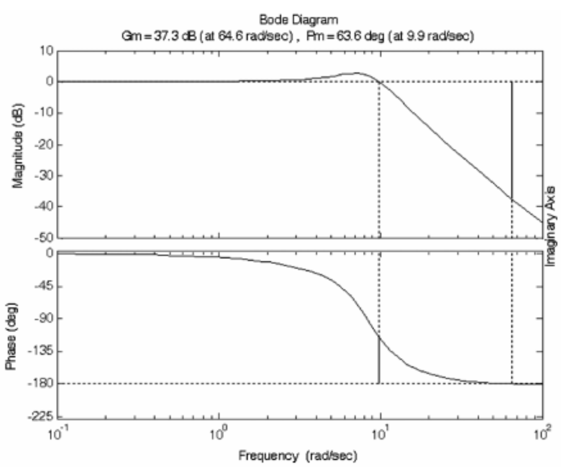


```
H3=[28.8  40560*k          0]
   [1     166.4+1352*k     0]
   [0     28.8        40560*k]
B3=[1 -496.6    27688.98*k]
   [1 166.4+1352*k       0]
   [0 28.8         40560*k]

J3 =[0.002(68.77*k^2+33.72*k+1)/(-0.34*k^2+k)]
K=0.0386
 Transfer function:

        54 s + 1620
 ----------------------------
 s^3 + 28.8 s^2 + 220.5 s + 1622.4

 G(m)=37.3Db(at 64.6 rad/sec)
 P(m)= 63.6 deg (at 9.9 rad/sec)

A =

  -28.8000  -13.7813  -12.6750
   16.0000         0         0
         0    8.0000         0

B =

   4
   0
   0

C =

   0   0.8438   3.1641

D = 0
rank_of_M =3
system_order = 3
rank_of_N =3

 num=[54 1620];
den=[1 28.8 220.5 1622.4];
 T=0.02;
num/den = G'

  0.010802 z^2 + 0.0049442 z - 0.0059662
  --------------------------------------
  z^3 - 2.4902 z^2 + 2.0622 z - 0.56214

a0= - 0.56214
a1= 2.0622
a2= - 2.4902
a3=1
b0=0.66
b1=-1.35
b2=-2.8

G'(1)>0
G'(-1) is negative
 1a31<1a01
1b21>1b01
```

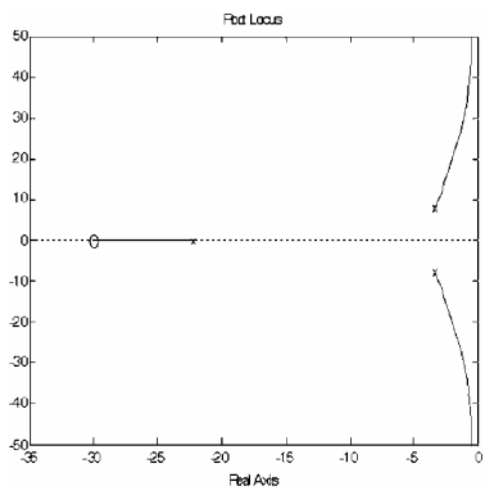
7.2. Results

GAIN MARGIN= 37.3Db, PHASE MARGIN = 63.6 deg (at 9.9 rad/sec), Rank of the controllability & observability matrix = 3, as same as the order of the system & hence controllable & observable. Jury stability criteria are satisfied.

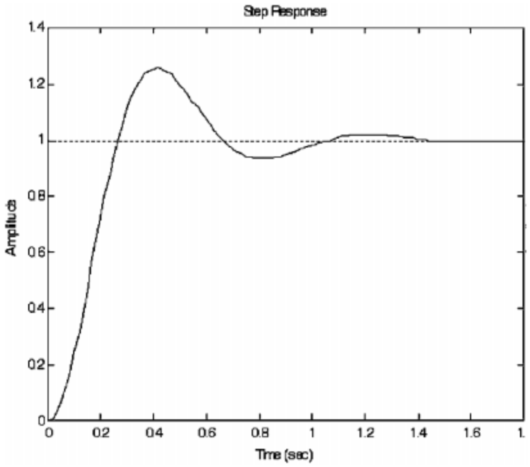
7.3. Matlab Output Plot



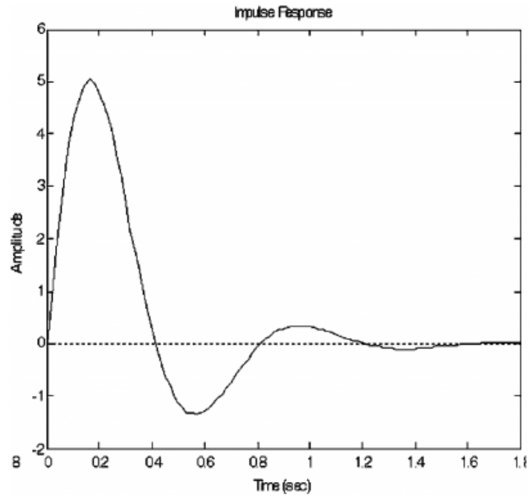
(a)



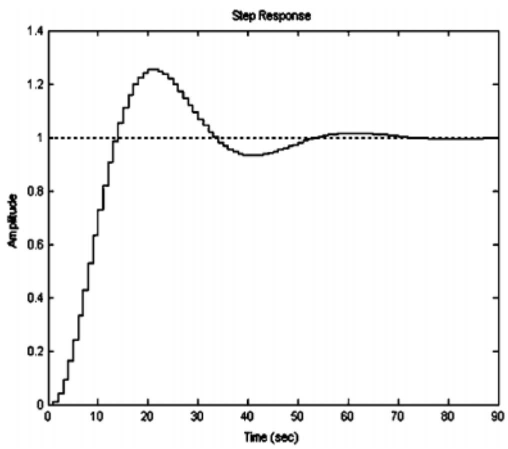
(b)



(c)



(d)



(e)

Fig.2: Matlab Output Plot Graph (a) Bode Plot; (b) Root Locus Plot; (c) Step Response Plot; (d) Impulse Response Plot; (e) Step Response Plot in Sample Domain

8. CONCLUSION

The system offers an indepth sight for the biomedical engineering application of a human heart rate control system operated in optimal condition. As it is analyzed using MATLAB software, the overall system is found to have gain margin of value 37.3 dB, phase margin of value 63.6 degree. Since the system is stable in both continuous and sample data control system, controllable, observable, having appropriate gain margin and phase margin, the design of an artificial heart rate control system becomes feasible as done in this present paper

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