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A NEW HYBRID IMAGE DENOISING METHOD

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A new model based on the hybridization of wavelet and bilateral filters for denoising of variety of noisy images is presented in this paper. The model is experimented on standard images, like x-ray images, ultrasound and astronomical telescopic images and the performances are evaluated in terms of peak signal to noise ratio (PSNR) and image quality index (IQI). Results demonstrate that use of bilateral filters in combination with wavelet thresholding filters on subbands of a decomposed image deteriorates the performance. But the application of bilateral filter before and after decomposition enhances the performance. The filter developed with bilateral filter before and after the decomposition of an image is found to be the best performance amongst four models considered in this study in terms of PSNR and IQI. In addition, the proposed filter gives nearly uniform and consistent results on all the images.

Keywords: Image Denoising, Wavelet Transform, Wavelet Thresholding, Bilateral Filter

1. Introduction

Now-a-days an image is synonymous to digital image and is very much essential for daily life applications such as satellite television, medical imaging (magnetic resonance imaging, ultrasound imaging, x-ray imaging), computer tomography. It is also essential for the researches in the areas of Science and Technology such as geographical information systems and astronomy. The images collected by different type of sensors are generally contaminated by different types of noises [1].

Digital images may be contaminated by different sources of noise. Noise may be generated due to imperfect instruments used in image processing, problems with the data acquisition process, and interference, all of which can degrade the data of interest. Furthermore, noise can be introduced by transmission errors and compression also [1]. Different types of noises are introduced by different noise sources like dark current noise is due to the thermally generated electrons at sensor sites. It is proportional to the exposure time and highly dependent on the sensor temperature. Shot noise, which has the characteristics of Poisson distribution, is due to the quantum uncertainty in photoelectron generation. Amplifier noise and quantization noise occur during the conversion of number of electrons to pixel intensities. The overall noise characteristics in an image depends on many factors, which include sensor type, pixel dimensions, temperature, exposure time, and ISO speed [7].

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Noise is also channel dependent. Typically, green channel is the least noisy while blue channel is the noisiest channel. That means noise is in general not white. Noise in a digital image has low as well as high frequency components. Though the high-frequency components can easily be removed, it is challenging to eliminate low frequency noise as it is difficult to distinguish between real signal and low-frequency noise. Most of the natural images are assumed to have additive random noise, which is modelled as Gaussian type. Speckle noise [4] is observed in ultrasound images, whereas Rician noise [5] affects MRI images. Thus, denoising is often a necessary and the first step to be considered before the image data is analyzed. It is necessary to apply an efficient denoising technique to compensate for any data corruption [1].The goal of denoising is to remove the noise while preserving the important image information as much as possible.

Linear filtering techniques, such as Wiener filter or match filter, have been used for this purpose for many years. But linear filters may result in some problems, such as blurring the sharp edges, destroying lines and other finer image details. They generally fail to effectively remove heavy tailed noise. Due to these facts, an alternative filtering technique like nonlinear filtering is necessary. Many works [8]-[10] have been reported on image denoising using nonlinear filters. Thresholding algorithm in an orthogonal transform domain, such as subband or wavelet transform, is a nonlinear filter. Subband transform with orthogonal perfect reconstruction filter-banks is an orthogonal transform. It is known that the sub-band filters act as a set of discrete time based functions in a vector space and the decomposition of signal is just to project the signal onto these base functions. As for a signal with noise, there are some differences between the coefficients of original signal and noise because of their different features. In general, if an orthogonal transform with high-energy compaction and de-correlation properties is used, most of the energy of the original signal will be compacted into a few high magnitude coefficients [11], [12]. If the image data is corrupted by additive white noise, components that correspond to noise will be distributed among low magnitude high frequency components. Most of the coefficients of noise are of smaller amplitudes. So, it is reasonable to eliminate the noise by comparing all the coefficients with a threshold and cutting off those coefficients with smaller values than the thresholds [6], [13].

Recently a lot of developments have been observed on the use of wavelet transforms not only in image processing but also in various fields of signal processing, because of their inherent capability to capture not only the different frequency components present in a signal but also their time (localization) of occurrence, using variable size time-windows for different frequency bands. This results in a high frequency resolution in low bands and low frequency resolution in high bands. Consequently, wavelet transform is a powerful tool for modeling non-stationary signals that exhibit slow temporal variations in low frequency and abrupt temporal changes in high frequency [3].

Many denoising methods have been proposed over the years, such as the Wiener filter, wavelet thresholding [6], anisotropic filtering [14], bilateral filtering [15], total variation method [16], and non-local methods [17]. Among these, wavelet thresholding has been reported to be a highly successful method. In wavelet thresholding, a signal is decomposed into approximation (low-frequency) and detail (high-frequency) subbands, and the coefficients in the detail subbands are processed via hard or soft thresholding [6], [13], [18], [19]. The hard thresholding eliminates (sets to zero) coefficients that are smaller than a threshold as discussed earlier; the soft thresholding shrinks the coefficients that are larger than the threshold as well. The main task of the wavelet thresholding is the selection of threshold value and the effect of denoising depends on the selected threshold: a bigger threshold will throw off the useful information and the noise components at the same time while a smaller threshold can not eliminate the noise effectively. Donoho [6] gave a general estimation method of threshold, but the best threshold cannot be found by this method. Chang et al. [20] have used predictive models to estimate the threshold. It is a spatially adaptive threshold based on context modeling. They also presented data-driven threshold for image denoising in a Bayesian framework [21]. In the SURE Shrink approach [19], the optimal threshold value based on the Stein's Unbiased Estimator for Risk (SURE) is estimated. A major strength of the wavelet thresholding is the ability to treat different frequency components of an image separately; this is important, because noise in real scenarios may be frequency dependent. But, in wavelet thresholding the problem experienced is generally smoothening of edges.

The bilateral filter was proposed in [15] as an alternative to wavelet thresholding. It applies spatially weighted averaging without smoothing edges. This is achieved by combining two Gaussian filters; one filter works in spatial domain, the other in the intensity domain. Therefore, not only the spatial distance but also the intensity distance is important for the determination of weights [7]. Hence, these types of filters can remove the noise in an image while retaining its edges. However, the filter may not be very efficient in removing any noise in the texture part of the image. It is not being able to remove salt and pepper type of noise. Also there is no theoretical works on optimization of the parameters of the filters.

In this work a hybrid denoising method is proposed to find the best possible solution, so that PSNR [22] and IQI [26] of the image after denoising are optimal. The proposed model is based on wavelet thresholding and bilateral filtering, which exploits the potential features of both wavelet thresholding and bilateral filter at the same time their limitations are overcome.

In view of the above the main objectives of the work are:

1. To develop a hybrid filter through the hybridization of wavelet thresholding and bilateral filters and to tune the different parameters of the filter to optimize the performance of the filter for denoising different types of images.
2. To tune the parameters of both the wavelet based filter and bilateral filters to optimize their performance for filtering the same types of images as in step (i).
3. To compare the performance of the filters developed in step (i) with those in step (ii) in denoising different types of images.

Section 2 of the paper introduces the concept of wavelet thresholding and works on it. Section 3 explains concepts of bilateral filtering along with its workings. Performance measurement criteria are discussed in section 4. Section 5 describes the proposed hybrid denoising model. Results are discussed in section 6. Finally the conclusions are drawn in section 7.

2. WAVELET DECOMPOSITION

The wavelet decomposition process involves three basic steps as follows:

i) a linear forward wavelet transform;

ii) nonlinear thresholding step and;

iii) a linear inverse wavelet transform.

2.1. Wavelet Representation of Image

Let $f = \{f_{ij}, i, j = 1, 2\ldots M\}$ denote the M × M matrix of the original image and M is some integer power of 2. During transmission the image f is corrupted by white Gaussian noise with independent and identically distributed (i.i.d) zero mean, and standard deviation σ i.e. $n_{ij} \sim N(0, \sigma^2)$. So, the noisy image received at the receiver end is $g_{ij} = f_{ij} + \sigma n_{ij}$. The goal is to estimate the signal f from noisy observations g_{ij} such that Mean Squared Error (MSE) is minimum and Peak Signal to Noise Ratio (PSNR) is maximum as well as image quality index(IQI) is also maximum within its range [0,1]. Let W and W^{-1} denote the two dimensional orthogonal discrete wavelet transform (DWT) matrix and its inverse respectively. Then Y = W.g represents the matrix of wavelet coefficients of g having four subbands (LL, LH, HL and HH). The sub-bands HH_k, HL_k, and LH_k are called details, where k is the scale varying from 1, 2 … J and J is the total number of decompositions. The size of the subband at scale k is $N/2^k \times N/2^k$. The subband LL_J is the low-resolution residue. The wavelet thresholding denoising method processes each coefficient of Y from the detail subbands with a soft threshold function to obtain $\hat{X}$ [23].

The denoised estimate is inverse transformed to $\hat{f} = W^{-1}\hat{X}$.

2.2. Wavelet Thresholding

It has been observed that in many signals energy is mostly concentrated in a small number of dimensions and the coefficients of these dimensions are relatively large compared to other dimensions or to any other signal (specially, noise) that has its energy spread over a large number of coefficients. Hence, in wavelet thresholding, each coefficient is thresholded (set to zero) by comparing against a threshold to eliminate noise, while preserving important information of the original signal [24]. Usually two types of thresholding techniques are used:

2.2.1. Hard Thresholding

The hard thresholding operator is defined as

$$D(U, \lambda) = U \text{ for all } |U| > \lambda$$
$$= 0 \text{ otherwise.}$$

Hard threshold is a "keep or kill" procedure and is more intuitively appealing.

2.2.2. Soft Thresholding

The soft thresholding operator is defined as

$$D(U, \lambda) = 0 \text{ for all } |U| \leq \lambda$$
$$= \operatorname{sgn}(U)\,(|U| - \lambda) \text{ otherwise.}$$

Soft thresholding shrinks the magnitudes of the coefficients above the threshold in absolute value and this method is used as the thresholding technique in this paper.

Determination of the value of the threshold is crucial as larger value may result into loss of information while smaller one may allow noise to continue.

The thresholding techniques have some underlying disadvantages. For instance, the estimated wavelet coefficients by the hard thresholding method are not continuous at the threshold ë which may lead to the oscillation of the reconstructed signal. In the soft thresholding case, there are deviations between image coefficients and thresholded coefficients which directly influence the accuracy of the reconstructed signal. Retention of the edges is also a problem here. Different edge detection algorithm are used to extract the contour feature of cell images. Bilateral filter may help to achieve the target of edge retention.

3. BILATERAL FILTER

Tomasi et al [15] proposed bilateral filter as an alternative to wavelet thresholding for image denoising. It applies spatial weighted averaging without smoothing edges. This is achieved by combining two Gaussian filters; one filter works in spatial domain, the other filter works in intensity domain. Therefore, not only the spatial distance but also the intensity distance is important for the determination of weights. At a pixel location x, the output of a bilateral filter can be formulated as follows:

$$\bar{I}(x) = \frac{1}{C}\Sigma_{y \in N(x)} e^{\frac{-\|y-x\|^2}{2\sigma_d^2}} e^{\frac{-1|1(y)-1(x)|^2}{2\sigma_r^2}} I(y) \tag{6}$$

where σ_d and σ_r are parameters controlling the fall-off of weights in spatial and intensity domains, N(x) is a spatial neighborhood of pixel I(x), and C is the normalization constant:

$$C = \Sigma_{y \in N(x)} e^{\frac{-\|y-x\|^2}{2\sigma_d^2}} e^{\frac{-|I(y)-I(x)|^2}{2\sigma_r^2}} \tag{7}$$

The bilateral filter is a special case of the Jacob algorithm. This single iteration of Jacob algorithm, which is known as the diagonal normalized steepest descent, yields the bilateral filter.

One weakness of the bilateral filter is its inability to remove salt-and-pepper type of noise. The second drawback of the bilateral filter is its single resolution nature. Unlike the wavelet filter, the bilateral filter may not access to the different frequency components of a signal. Although it is effective in removing high-frequency noise, the bilateral filter fails to remove low-frequency noise. Another crucial issue with the bilateral filter is that there is no theoretical work on the optimal values of σ_d and σ_r which are the parameters that control the behavior of the bilateral filter. Referring to eqn. (6), σ_d and σ_r characterizes the spatial and intensity domain behaviors, respectively. Although these parameters should be related to the noise and image characteristics, the issue has not been studied yet.

In this work, empirical study is carried out to find the optimal parameter values. White Gaussian noise is added to the images and are denoised by applying the bilateral filter with different values of the parameters σ_d and σ_r for evaluating the performance of the filter.

4. Measurement of Performance

To judge the performance of the denoising techniques Mean Squared Error (MSE) and Peak Signal to Noise Ratio (PSNR) [22].are the automatic choice for the researchers. But a better PSNR does not imply that the visual quality of the image is good. To overcome this problem Image Quality Index (IQI) [26] is considered in this work as the second parameter for judging the quality of denoised images. Although the index is mathematically defined and does not explicitly employ the human visual system model, experiments on various image distortion types show that it exhibits surprising consistency with subjective quality measurement.[2].

Let $f = \{f_i \mid i = 1,2,\ldots,M\}$ and $g = \{g_i \mid i=1,2,\ldots,M\}$ be the original and the denoised images respectively.

MSE may be defined by eqn. (8).

$$MSE = \frac{1}{M}\sum_{i=1}^{M}(g_i - f_i)^2 \tag{8}$$

where M is the number of elements in the image.

For example, if we wanted to find the MSE between the denoised and the original image, then we would take the difference between the two images pixel-by-pixel, square the results, and average the results.

The PSNR is defined by eqn. (9) as given below:

$$PSNR = 10\log_{10}\left(\frac{(2^n - 1)^2}{MSE}\right) \tag{9}$$

where n is the number of bits per symbol.

The Image Quality Index (IQI), Q, is defined by eqn. (10) as a product of three factors: loss of correlation, luminance distortion, and contrast distortion as

$$Q = \frac{\sigma_{fg}}{\sigma_f \sigma_g}\cdot\frac{2\bar{f}\,\bar{g}}{\bar{f}^2 + \bar{g}^2}\cdot\frac{2\sigma_f \sigma_g}{\sigma_f^2 + \sigma_g^2} \tag{10}$$

where

$$\bar{f} = \frac{1}{M}\sum_{i=1}^{M} f_i, \qquad \bar{g} = \frac{1}{M}\sum_{i=1}^{M} g_i$$

$$\sigma_f^2 = \frac{1}{M-1}\sum_{i=1}^{M}(f_i - \bar{f})^2, \quad \sigma_g^2 = \frac{1}{M-1}\sum_{i=1}^{M}(g_i - \bar{g})^2$$

$$\sigma_{fg} = \frac{1}{M-1}\sum_{i=1}^{M}(f_i - \bar{f})(g_i - \bar{g})$$

The first component of eqn.(10) represents the correlation coefficient between f and g, which measures the degree of linear correlation between f and g and its dynamic range is from –1 to 1. The second component, with a value range of [0,1], measures how close the mean luminance is between f and g. σ_f and σ_g can be viewed as the estimates of the contrast of f and g, so the third component with a value range of [0,1] measures how similar the contrasts of the images are.

Thus, Q can be rewritten as

$$Q = \frac{4\sigma_{fg}\bar{f}\,\bar{g}}{(\sigma_f^2 + \sigma_g^2)(\bar{f}^2 + \bar{g}^2)} \tag{11}$$

The dynamic range of Q is [–1, 1]. The best value 1 is achieved, if and only if, $g_i = f_i$ for all i = 1,2,...,M. The lowest value –1 occurs when $g_i = 2\bar{f} - f_i$ for all i = 1,2,…,M.

5. Proposed Approach

In this work a new model is proposed by hybridizing bilateral and wavelet based principles for image denoising. The performance of the model is compared with wavelet based denoising method[25], bilateral filtering based image denoising method [15] and the method proposed by Ming Zhang and Bahadur Gunturk (Zhang-Gunturk method) [7].

While working with the models, the parameters w, σ_d and σ_r of bilateral filters are varied over a wide range of values as there is no explicit rules that can guide the tuning of these parameters. And the threshold value for the wavelet based filter is also varied.

The PSNR is calculated by eqn. (9) and IQI by eqn. (11) for all the considered models with the considered parameters as discussed above.

The first model is developed with wavelet based thresholding algorithm. In this algorithm, the images are decomposed into four subbands. The soft thresholding method is then employed on all the subbands by varying the thresholding values from 0.001 to 0.1 and the best result is found with the threshold value as 0.01 using db8 filters available in Matlab. As most of the researchers have used db8 filters for image denoising, the same has been considered in this work too. Though lowering of the thresholding value below 0.01 yields better PSNR but the visual qualities of the denoised images are not as good as with the threshold value of 0.01.

The second model is the bilateral filter. The parameters, w, σ_d and σ_r are tuned for finding the optimal performance.

The parameter, σ_d is varied from 0.01 to 2.2, the window size, w, is varied from 1 to 11, and σ_r is varied from 10 to 70.

The third one is a hybrid model proposed by Ming Zhang and Bahadur Gunturk. As proposed by the authors, the images are decomposed into four subbands and both the wavelet based filter and bilateral filter are used.

Proposed model is the newly designed hybridized one as shown in figure 1. In this model, the image is denoised first with bilateral filter followed by decomposition into four subbands using db8 filters. In the next level the wavelet based soft thresholding is applied on all the subbands. The value of soft-thresholding is set at the same value obtained for model no. 1 with wavelet thresholding only. The results obtained after thresholding are then used to reconstruct the image. In the last level, again bilateral filter is applied to get the final denoised image.

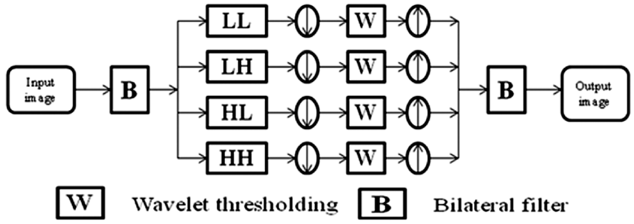

Fig. 1: The Proposed Hybrid Model

6. RESULTS & DISCUSSIONS

All the four models are experimented with standard pictures like Zelda, Baboon, x-ray image of arteries named as X-ray, ultrasound image like the tendon of the arm named as Ultrasound, 16 weeks foetus in uterus called Fetus, astronomical telescopic image Astro. The PSNR and IQI values obtained for all the models considering all the images are given in Table 1 and Table 2 respectively.

Table 1
Performance of the Considered Models in Terms of PSNR

Models ↘ Images->	Baboon	Zelda	Fetus	Ultrasound	X-ray	Astro
Model1	40.5473	40.4891	40.4898	40.1451	40.5623	40.5527
Model2	45.6642	45.1392	43.765	42.8462	42.3595	42.1762
Model3	23.7287	17.6064	25.6703	22.8115	26.377	6.2564
Model4	47.8465	47.9035	46.3011	45.7466	45.5265	45.0764

Table 2
Performance of the Considered Models in Terms of IQI

Models ↘ Images->	Baboon	Zelda	Fetus	Ultrasound	X-ray	Astro
Model1	0.9784	0.9849	0.9823	0.9981	0.9634	0.9699
Model2	0.9967	0.9924	0.9837	0.976	0.9729	0.9675
Model3	0.7487	0.8642	0.9414	0.0379	0.9358	0.0603
Model4	0.9956	0.9967	0.9917	0.9889	0.9885	0.985

The figures 2 and 3 depict the comparative performance of the four models in terms of the quality parameters, PSNR and IQI respectively.

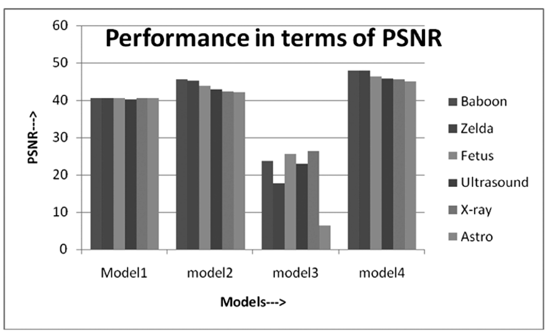

Fig. 2: Performance Comparison in Terms of PSNR

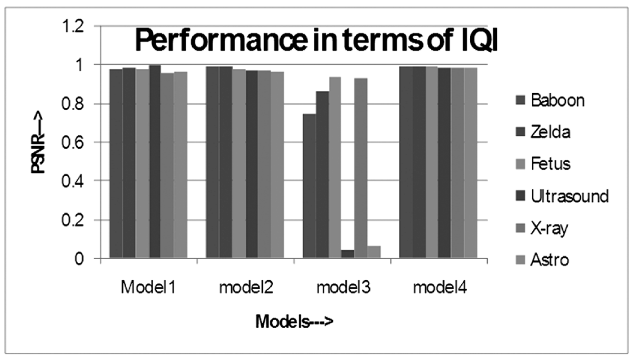

Fig. 3: Performance Comparison in Terms of IQI

The proposed model is found to be well capable in denoising all the images as compared to other models.

Among the other models, model 2 is very close to the proposed model in terms of PSNR but in terms of IQI, models 1 and 2 both are working closely with the proposed one.

The images that are considered for the experimentation are represented in the charts by different bands to give an easy and smooth visual effects.

The original images along with the noisy ones and the ultimate denoised images by the proposed model are presented as a, b, c respectively in Figures 4 to 7

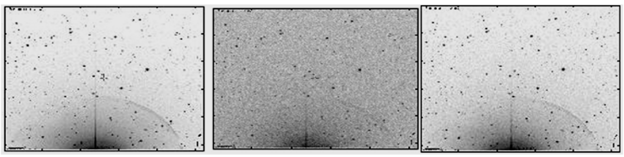

Fig. 4: Astronomical Image: a) Original Image b) Noisy Image c) Denoised Image

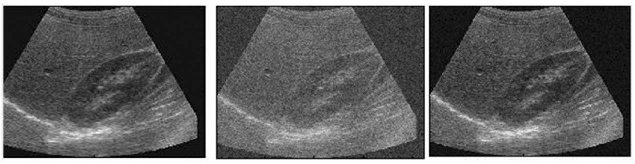

Fig. 5: USG Image: a) Original Image b) Noisy Image c) Denoised Image

Fig. 6: Standard Image: a) Original Image b) Noisy Image c) Denoised Image

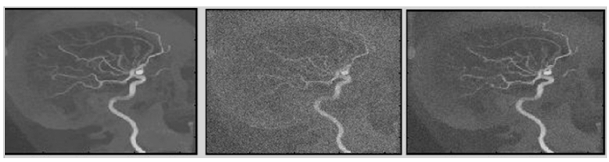

Fig. 7: X-ray Image: a) Original Image b) Noisy Image c) Denoised Image

Observation of the results reveals that when only the wavelet based thresholding filter is used on all the decomposed subbands of any image, then it results in good PSNR and IQI. But when bilateral filter is applied in different ways along with wavelet based thresholding filters on the subbands as proposed by Zhang et al [7], the performance deteriorates. However, when the bilateral filter is used on the image before decomposition and also on the reconstructed image after decomposition, the performance improves.

7. CONCLUSIONS

A hybrid denoising model is designed through hybridization and its performance is tested on different types of noisy images. Comparison of the results is drawn along with other three methods using established wavelet and bilateral based filtering techniques. The performances of the models are evaluated in terms of PSNR and IQI and comparison is drawn. It has been observed that out of the four models the performance of the proposed model (model 4) is the best amongst four in terms of PSNR and IQI, in addition to be more uniform and consistent in all the types of images tested with. It is also observed that the application of bilateral filters on wavelet decomposed subbands in any combination with wavelet thresholding deteriorates the performance of the model, whereas, the application of bilateral filters on both before and after decomposition enhances the performance. Thus, this model is recommended as a well competent and efficient model for denoising any type of images.

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