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AN INVENTORY MODEL FOR DETERIORATING ITEMS WITH ON HAND INVENTORY DEPENDENT RAMPED TYPE DEMAND RATE

D. Samal[1], A. Kalam[2], S.K.Sahu[3] & M. Mishra[4]

This model discuss an infinite time horizon deterministic inventory model for deteriorating items without shortages where the demand rate declines along with stock level down to a certain level of inventory and also the demand rate becomes constant for the rest of the cycle and the deterioration is a ramp type function. The decision rule for finding the optimal order quantity is also given. Five numerical examples are presented to illustrate the model developed.
Keywords: Inventory, Shortage, Demand, Deterioration, Ramp.

Introduction

The classical inventory models consider the demand rate to be either constant or to be time dependent but independent of the stock status. However for certain types of inventory, particularly consumer goods, and the consumption rate may be influenced by the stock levels, that is, the consumption rate may go up or down with the on-hand stock level. As reported by Levin et al [1972] and Silver and Peterson [1985], sales at the retail level tend to be proportional to inventory displayed and a large piles of goods displayed in a supermarket will lead the customers to bye more. These observations have attracted many marketing researchers and practioners to investigate the modeling aspects of this phenomenon. Gupta and Vrat [1986] first developed a model for consumption environment to minimize the cost with the assumption that stock- dependent consumption rate is a function of the initial stock level. Silver and Meal [1973], Datta and Pal [1988] has progressed the model where demand rate is not required to be constant.

Among the important paper published so far with inventory-level-dependent demand rate, mention may be made of by Gupta and Vrat [1986], Mandal and Phaujdar [1989], baker and Urban [1988] etc. Gupta and Vrat[1986] have discussd a situation where the demand rate has been assumed to dependent on the order quantity whereas, Mandal and Phaujdar[1989] have discussed an inventory level. Baker and Urban[1988] have analyzed a similar situation assuming the demand rate to be dependent on the on-hand inventory according to the relation $R(i) = \alpha i^{\beta}$ where $\alpha > 0, 0 < \beta < 1$. Sahu et al have analysed a similar situation, assuming the demand rate to be depend on the on-hand inventory i according to the relation $R(i) = \alpha e^{-\beta i}$ where $\alpha > 0, 0 < \beta < 1$.

The present inventory model makes an attempt to study the situation where the demand rate decline along with stock level down to a certain level of inventory and also the demand rate become constant for the rest of the cycle. It is well known that the stock level has a motivation affect on the customers but the experience shows that customers arrive to purchase goods attracted by huge stockdown to a certain level of declining inventory. The only a limit numbers of customer arrive to purchase goods owing to such factors goodwill, good quantity, genuine price level of the goods, locality of the shop, good quality of the items etc. The present model is applicable in such a situation when the β approaches to zero.

Fundamental Assumptions and Notations of the Model

The basic assumption made to develop the proposed model are the following:

1. Replenishment rate is infinite i.e, replenishment is instantaneous but replenishment size is finite.
2. Lead time is zero.
3. No shortages are permitted.
4. The selling price s, unit cost C, holding cost C_1 per unit per unit time and replenishment cost C_3 per replenishment are known and constant.
5. The time horizon is finite.
6. The inventory system involves only one item.
7. The demand rate is dependent on the on-hand inventory down to a level S_0, beyond which it is

[1]Department of Mathematics, College of Engineering Bhubaneswar, Orissa
[2]Department of Mathematics, MIT, Taraboi, Khurda-752055, Orissa
[3]Department of Mathematics, Govt Science College Malkanagiri, Orissa
[4]NIFM, Sector-48, Pali Road Faridabad-121001, Haryana
Email: abdul_003m@rediffmail.com

assumed to be constant. The demand rate R(i) of the item when the on hand inventory level is i

$$R(i) = \alpha e^{-\beta i}, \quad i \geq S_0$$

$$R(i) + Hi(t) = D, \; 0 \leq i \leq S_0, \; \alpha > 0, \; 0 < \beta < 1,$$

$$D = \alpha e^{-\beta S_0}$$

$$\text{Where } H = \begin{cases} 0, \text{when} 0 < t < t_1 \\ \theta, \text{when} t > t_1 \end{cases}, \qquad 0 < \theta < 1$$

BASIC EQUATIONS GOVERNING THE MODEL AND THEIR SOLUTIONS

The basic equations governing the present model are the following:

$$\frac{di}{dt} = -\alpha e^{-\beta i}, \qquad 0 \leq t \leq t_1 \tag{1}$$

$$\frac{di}{dt} + \theta i(t) = -D, \; t_1 \leq t \leq T \tag{2}$$

Where $t, i, \alpha, \beta, D, t_1, T$ are defined.

The solution of the differential equation (1) using the initial condition i = s at t = 0, is

$$i = \frac{\ln\left(-\alpha\beta t + e^{\beta S}\right)}{\beta}, \qquad 0 \leq t \leq t_1 \tag{3}$$

Using the condition $i = S_0$ at , $t = t_1$,

$$S_0 = \frac{\ln\left(-\alpha\beta t_1 + e^{\beta S}\right)}{\beta} \tag{4}$$

$$\text{So } t_1 = \frac{e^{\beta S} - e^{\beta S_0}}{\alpha\beta} \tag{5}$$

The solution of the differential equation (2) using the condition i = 0 at t = T is

$$i = \frac{D}{\theta}\left\{e^{\theta(T-t)} - 1\right\} \qquad t_1 \leq t \leq T \tag{6}$$

Using the condition $i = s_0$ at $t = t_1$, we find from (6)

$$S_0 = \frac{D}{\theta}\left\{e^{\theta(T-t_1)} - 1\right\}$$

$$t_1 = T - \frac{1}{\theta}\ln\left(\frac{\theta S_0}{D} + 1\right) \tag{7}$$

From (5) and (7) we get

$$T = \frac{e^{\beta S} - e^{\beta S_0}}{\alpha\beta} + \frac{1}{\theta}\ln\left(\frac{\theta S_0}{D} + 1\right) \tag{8}$$

$$\text{Now } H = \int_0^T idt = \int_0^{t_1} idt + \int_{t_1}^T idt$$

Substituting the value of I in the above integrals and then integrating, we find

$$H = \int_0^{t_1} \frac{\ln\left(-\alpha\beta t + e^{\beta S}\right)}{\beta} dt + \int_{t_1}^T \frac{D}{\theta}\left\{e^{\theta(T-t)} - 1\right\}$$

Eliminating t_1 and T

$$H = \frac{\theta^3 e^{\beta S}(\beta S - 1) - \theta^3 e^{\beta S_0}(\beta S_0 - 1) + S_0\theta^2\alpha\beta^2 + D\alpha\beta^2 \ln\left(\frac{\theta S_0}{D} + 1\right)}{\alpha\beta^2\theta^3} \tag{9}$$

Now the profit function $\pi(s)$ (profit per unit time) is

$\pi(S)=$

$$\frac{(s-C)S\alpha\beta^2\theta^3 + C_1\left\{\theta^3 e^{\beta S}(\beta S - 1) - \theta^3 e^{\beta S_0}(\beta S_0 - 1) + S_0\theta^2\alpha\beta^2 + D\alpha\beta^2 \ln\left(\frac{\theta S_0}{D} + 1\right)\right\}}{\beta\theta^3\left[e^{\beta S} - e^{\beta S_0}\right] + \alpha\beta^2\theta^2 \ln\left(\frac{\theta S_0}{D} + 1\right)} \tag{10}$$

The necessary condition for $\pi(S)$ to be a maximum is

$\frac{d\pi(S)}{dS} = 0$ gives

$$\begin{aligned} & e^{\beta S}\{\alpha\beta^3\theta^6(s-C) + C_1\theta^6\beta^2 e^{\beta S_0}(\beta S_0 - 1) - C_1\theta^5 S_0\alpha\beta^4 \\ & -C_1 D\alpha\beta^4\theta^3 \ln\left(\frac{\theta S_0}{D} + 1\right) + C_3\alpha\beta^4\theta^6\} \\ & +Se^{\beta S}\{C_1\beta^3\theta^6 e^{\beta S_0} - C_1\alpha\beta^3\theta^3 \ln\left(\frac{\theta S_0}{D} + 1\right) \\ & -\alpha\beta^4\theta^6(s-C)\} - e^{2\beta S}\left(C_1\beta^2\theta^6\right) \\ & +\alpha\beta^3\theta^3(s-C)\ln\left(\frac{\theta S_0}{D} + 1\right) = 0 \end{aligned} \tag{11}$$

$$T = \frac{e^{\beta S} - e^{\beta S_0}}{\alpha\beta} + \frac{1}{\theta}\ln\left(\frac{\theta S_0}{D} + 1\right) \tag{12}$$

The roots of the equation (11) give the global maximum for the profit function $\pi(S) = \pi(S^*)$ occurs at $S = S^*$ and $T + T^*$. Such a positive root S of equation (11) for which $d^2\pi(S) / dS^2 < 0$ gives a local maximum for the profit function $\pi(s)$.

NUMERICAL EXAMPLES

To illustrate the model developed, the following five numerical examples have been considered.

Example = 1: We choose $\alpha = 0.5$, $\beta = 0.4$, $\theta = 0.01$, $s = 20$, $C = 10$, $S_0 = 10$, $C_3 = 10$, $C_1 = 0.5$

Using the decision rule, we get $T^* = 21.6$ year and $\pi(S^*)$ = \$502.639/year.

Example=2: We choose $\alpha = 5$, $\beta = 0.4$, $\theta = 0.01$, $s = 20$, $C = 10$, $S_0 = 28$, $C_3 = 10$, $C_1 = 0.5$

Using the decision rule, we get $T^* = 26.39$ year and $\pi(S^*) = \$729.328$/year.

Example=3: We choose $\alpha = 0.5$, $\beta = 0.4$, $\theta = 0.01$, $s = 20$, $C = 10$, $S_0 = 6$, $C_3 = 10$, $C_1 = 0.5$.

Using the decision rule, we get $T^* = 16.369$ year and $\pi(S^*) = 329.158$/year.

Example = 4: We choose, $\alpha = 0.5$, $\beta = 0.4$, $\theta = 0.01$, $s = 20$, $C = 10$, $S_0 = 15$, $C_3 = 10$, $C_1 = 0.5$

Using the decision rule, we get $T^* = 18.23$ year and $\pi(S^*) = \$419.23$/year.

Example = 5: We choose, $\alpha = 5$, $\beta = 0.4$, $\theta = 0.01$, $s = 20$, $C = 10$, $S_0 = 15$, $C_3 = 10$, $C_1 = 0.5$

Using the decision rule, we get $T^* = 21.9$ year and $\pi(S^*) = \$623.89$/year.

CONCLUSION

The current inventory model contains two types of demand rate of an item varies due to stock level in first type of demand and constant demand in second type of demand. The demand rate is dependent in the initial stock level are analyzed as are models in which demand rate is dependent. On the instantaneous stock level, the stock level has a motivational effect on the customers.

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