International Journal of Information Technology and Knowledge Management
July-December 2010, Volume 2, No. 2, pp. 539-543

Rough Definability on Knowledge Representation Systems

Debadutta Mohanty

The concept of different types of classifications with respect to rough definability are produced through algebraic approach. We determine some properties on the types of definability upon union, intersection and complementation. Also the types where the De Morgan's laws do hold are established.

Keywords: Rough set, knowledge, algebraic approach, De Morgan's Laws.
Subject Classification: 68 T 30, 92 J 40

1. Introduction

Most of our traditional tools for formal modeling, reasoning and computing are crisp, deterministic and precise in character. Real situations are very often not crisp and deterministic and they cannot be described precisely. For a complete description of a real system often one would require by far more detailed data than a human being could ever recognize and understand. This observation led to the extension of the concept of crisp sets so as to model imprecise data which can enhance their modeling power.

Z. Pawlak [2] introduced rough set theory (1982) which is a new mathematical tool dealing with vagueness and uncertainty. Both rough sets and fuzzy sets are the theories to handle uncertain, vague, imprecise problems but their view points are different. Fuzzy set theory was defined by L. Zadeh [5] in 1965. Many researchers have made much work in combining both of them, the results are rough fuzzy sets and fuzzy rough sets.

Algebraic approach to rough sets was introduced by T. Iwinski [1] in 1987. In a note Y.Yao ([6]) compares constructive and algebraic approaches in the study of rough sets. In the constructive approach one can define a pair of lower (inner) and upper (outer) approximation operators using the binary relation. In the algebraic approach, one defines a pair of dual approximation operators and states axioms that must be satisfied by the operators. Various classes of rough set algebras are characterized by different sets of axioms.

Rough set theory was introduced and developed by Z. Pawlak in 1982 ([2]). Rough set is being used as an effective model to deal imprecise knowledge. One of the main goals of the rough set analysis is to synthesize approximation of concepts from the acquired data. According to Pawlak, knowledge about a universe can be considered as one's capability to classify objects of the universe. By classification or partition of a universe U we write ,a set of objects $\{Y_i, i = 1,2,3 \ldots\ldots, n\}$ of U $Y_i \cap Y_j = \varphi$ for $i \neq j$ and $\bigcup_{i=1}^{n} Y_i = U$. Relation $R \subseteq U \times U$ be an equivalence relation on U. The equivalence class of an element $x \in U$ with respect to R, denotes $[x]_R$ is the set of elements $x \in U$ such that xRy. The set U/R be the family of all equivalence classes of R called as concepts or categories of knowledge R and $[x]_R$ a category in U. We know that the concepts of an equivalence relation R and classification $\{Y_i\}$ are mutually interchangeable The R be knowledge on U which is an equivalence relation on U.

Let U is non empty finite set called the universe of discourse and R be an equivalence relation over U. Given any arbitrary set $B \subseteq U$ it may not be possible to describe B precisely in the approximation space (U, R).The set B be characterized by a pair of approximation sets. This leads to the concept of rough set.

We define

$\underline{R}B = \cup \{y \in U/R : y \subseteq B\} = \{ x \in U : [x]_R \subseteq B\}$ and

$\overline{R}B = \cup \{y \in U/R : y \cap x \neq \varphi\} = \{x \in U : [x]_R \cap B \neq \varphi\}$

are called R-lower approximation and R-upper approximation of X, respectively, of B with respect to R.

The R-boundary region of B be denoted by $BN_R(B)$ defined by $BNR(B) = \overline{R}B - \underline{R}B$. We say that X is rough with respect to R if and only if $\overline{R}B \neq \underline{R}B$ and B is said to be R-definable if and only if $\overline{R}B = \underline{R}B$, that is $BN_R(B) = \varphi$, at that time B becomes a crisp set.

The set $\underline{R}B$ consists of all those elements of U which can be classified as the elements of U which can be classified as the elements of B with certainty, employing the knowledge R. The set $\overline{R}B$ consists of all those elements of U which can be classified as element of B employing the knowledge R. The set $BN_R(B)$ is the set of elements which can not be classified as either belonging to B or belonging to ~B having the knowledge R.

Department of mathematics, Seemanta Mohavidyalaya, Jharpokhoria, Mayurbhanj, 757086
Email: mohanty.deba@yahoo.com

We say that B is said to be rough (or R-Rough) if and only if $\bar{R}B \neq \underline{R}B$ that is $BN_R(B) \neq \varphi$ otherwise B is said to be R-definable and at that time $BN_R(B) = \varphi$.

This becomes Pawlak's definition (constructive method) of rough set ([3]). The system (2^U, $\cup$, $\cap$, $\bar{R}$, $\underline{R}$) is called rough set algebra, where $\cap$, $\cup$, ~ are set union, intersection and complement respectively. The lower and upper approximation in (U, R) have the following properties ; for any subset X, Y $\subset$ U,

1.1 $\underline{R}(X) \subseteq X \subseteq \bar{R}(X)$

1.2 $\underline{R}U = \bar{R}U = U, \underline{R}\varphi = \bar{R}\varphi = \varphi$

1.3 $\bar{R}(X \cup Y) = \bar{R}(X) \cup \bar{R}(Y)$ and

$\underline{R}(X \cap Y) = \underline{R}(X) \cap \underline{R}(Y)$

1.4 $\bar{R}(X \cap Y) \subseteq \bar{R}(X) \cap \bar{R}(Y)$ and

$\underline{R}(X \cup Y) \supseteq \underline{R}(X) \cup \underline{R}(Y)$

1.5 $\bar{R}(-X) = -\underline{R}(X), \underline{R}(-X) = -\bar{R}(X)$

1.6 $\underline{R}(\underline{R}(X)) = \bar{R}(\underline{R}(X)) = \underline{R}(X), \bar{R}(\bar{R}(X)) =$

$= \underline{R}(\bar{R}(X)) = \bar{R}(X)$

Algebraic or axiomatic definition to rough set was given by T.Iwinski [1] in 1987. Let P, Q be two sets such that $P \subset Q \subset U$. Then the pair (P, Q) form a rough set for which P be the below (Lower) and Q be the above (Upper) approximation concept. Applying some operational criteria to P, Q it can be converted to Pawlak's Rough set $(\bar{R}B, \underline{R}B)$, $B \subset U$.

Throughout this paper we use the algebraic definition of rough set.

2. DEFINITION ALGEBRAICALLY

Definition 2.1: ([6])

Let U be a finite and non empty set, called the universe. Let L , H : $2^U \rightarrow 2^U$ are two unary operators on the power set 2^U of U. These two operators are dual if

2.1 L ~ A = ~HA

2.2 H ~ A = ~LA for all $A \subseteq U$

Definition 2.2 : ([6])

Let L, H : $2^U \rightarrow 2^U$ are dual unary operators which satisfy

2.3 LU = U

2.4 $H\varphi = \varphi$

2.5 $L(A \cap B) = LA \cap LB$

2.6 $H(A \cup B) = HA \cup HB$, where A, B are the subsets of U

Also L and H satisfy the weaker conditions

2.7 $L(A \cup B) \supseteq LA \cup LB$

2.8 $H(A \cap B) \subseteq HA \cap HB$

2.9 $A \subseteq B \Rightarrow LA \subseteq LB$

2.10 $A \subseteq B \Rightarrow HA \subseteq HB$

Definition 2.3

Let L, H : $2^U \rightarrow 2^U$ be a pair of dual unary operators which satisfy (2.3) to (2.6) of the definition 2.2 and $LA \subset A \subset HA$, A be an element of 2^U, then the system $(2^U, \cup, \cap, \sim, L, H)$ be called a rough set algebra , where L, H are called lower and upper approximation operators respectively and the pair (LA, HA) form a rough set of A on U.

Definition 2.4 Let $\mathbb{A}$ = (LA, HA), $\mathbb{B}$ = (LB, HB) be two rough sets of A and B respectively on U, then the union, denotes $\mathbb{A} \cup \mathbb{B}$ and the intersection, denotes $\mathbb{A} \cap \mathbb{B}$, be defined by $\mathbb{A} \cup \mathbb{B}$ = (L(A $\cup$ B), H (A $\cup$ B)) and $\mathbb{A} \cap \mathbb{B}$ = (L(A $\cap$ B), H(A $\cap$ B)) respectively.

Definition. 2.5: Let $\mathbb{A}$ = (LA, HA), $\mathbb{B}$ = (LB, HB) be two rough sets defined on U then the subset, $\mathbb{A} \subset \mathbb{B}$ is defined by $LA \subset LB$ and $HA \subset HB$. Two rough sets $\mathbb{A}$, $\mathbb{B}$ are equal if and only if $\mathbb{A} \subset \mathbb{B}$ and $\mathbb{B} \subset \mathbb{A}$.

If $\mathbb{A}$ = (LA, HA) be a rough set of A on U then the complement of denotes ~ $\mathbb{A}$, be defined by ~ $\mathbb{A}$ = (~ HA, ~ LA) where ~LA, ~HA are respective complement of LA and HA in U.

Proposition 2.1 For any three rough sets $\mathbb{A}$, $\mathbb{B}$ and $\mathbb{D}$

(i) $\mathbb{A} \cup \mathbb{B} = \mathbb{B} \cup \mathbb{A}$ (ii) $\mathbb{A} \cap \mathbb{B} = \mathbb{B} \cap \mathbb{A}$

(iii) $\mathbb{A} \cup (\mathbb{B} \cup \mathbb{D}) = (\mathbb{A} \cup \mathbb{B})$ (iv) $(\mathbb{A} \cup \mathbb{B}) \cap \mathbb{D} = \mathbb{A} \cap (\mathbb{B} \cap \mathbb{D})$

(v) $\mathbb{A} \cap (\mathbb{B} \cup \mathbb{D}) = (\mathbb{A} \cap \mathbb{B}) \cup (\mathbb{B} \cap \mathbb{D})$

Now we find the theorems in which equalities hold for inclusion (2.7) and (2.8). We use the symbol $\subsetneq$ for subset with not equal to.

Theorem 2.1: Let U be a universe and {E_1, E_2 ………… E_n} be a partition of U. Let L, H: $2^U \rightarrow 2^U$ be two dual unary operators satisfying (2.3) to (2.6) of definition (2.2). Then for any two subsets X, Y $\subseteq$ U,

$LX \cup LY \subsetneq L(X \cup Y)$ if and only if there exist at least one $E_j (1 \leq j \leq n)$ such that

2.11 $X \cap E_j \subsetneq E_j$, $Y \cap E_j \subsetneq E_j$ and

2.12 $X \cup Y \supseteq E_j$ for $1 \leq j \leq n$

Proof : First suppose that, $X \cap E_j \subsetneq E_j$ for j = 1,2, ….. n, that is, $E_j \not\subset X$ and $E_j \not\subset Y$.

This implies $E_j \not\subset LX$ and $E_j \not\subset LY$ $1 \leq j \leq n$

But $E_j \subseteq L(X \cup Y)$. Thus $LX \cup LY \subsetneq L(X \cup Y)$

Conversely, suppose that $LX \cup LY \subsetneq L(X \cup Y)$

Then there exists one $E_j \subset U$, $j = 1, 2, \dots n$ such that $E_j \subset L(X \cup Y)$ but $E_j \not\subset LX \cup LY$, that is, $E_j \subset X \cup Y$ and $E_j \not\subset X$, $E_j \not\subset Y$.

Hence $X \cup Y \supseteq E_j$ and $X \cap E_j \subsetneq E_j$, $Y \cap E_j \subsetneq E_j$, for $1 \le j \le n$.

Theorem 2.2 : Let U be a finite universe and $E_1, E_2, \dots\dots E_n$ be a partition of U.

Let $L, H : 2^U \to 2^U$ be two dual unary operators satisfying (2.3) – (2.6) of definition 2.2.

Then for any two subsets $X, Y \subseteq U$, $LX \cup LY = L(X \cup Y)$

If and only if there exists no E_j, $1 \le j \le n$, such that the condition (2.11) and (2.12) do hold.

We find theorems on equality for Pawalak rough set as:

Corollary 2.1: Let $R : U \to U$ be an equivalence relation and let $E_1, E_2, \dots\dots E_n$ be a partition of U under R. Then for any two subsets X, Y of U, $\underline{R}X \cup \bar{R}Y \subsetneq (X \cup Y)$,

If and only if there exists atleast one E_j, $1 \le j \le n$, such that and $X \cup Y \supseteq E_j$ hold.

Corollary 2.2: Let $R : U \to U$ be an equivalence relation and let $E_1, E_2, \dots\dots E_n$ be a partition of U under the relation R. Then for two subsets $X, Y \subseteq U$.

$\underline{R}X \cup \underline{R}Y = \underline{R}(X \cup Y)$, if and only if there exists no E_j, $1 \le j \le n$, such that $X \cap E_j \subsetneq E_j$, $Y \cap E_j \subsetneq E_j$ and $X \cup Y \supseteq E_j$ hold.

The proof follows directly by taking $\underline{R}X$ instead of LX, and $\underline{R}Y$ instead of LY and $\underline{R}(X \cup Y)$ instead of $L(X \cup Y)$.

Note 3: According to Pawlak ([3]), dividing the knowledge base into smaller fragments causes loss of information. Here we find the conditions for which there is no loss of information though it is distributed among the smaller fragments.

Theorem 2.3 : Let $E_1, E_2, \dots\dots E_n$ be a partition of a finite universe U.

Let $L, H : 2^U \to 2^U$ be two dual unary operators satisfying (2.3) – (2.6) of definition 2.2., then for any two subsets $X, Y \subseteq U$, $H(X \cap Y) \subsetneq HX \cap HY$.

if and only if there exists at least one E_j, $1 \le j \le n$, such that

2.13 $\quad X \cap E_j \ne \varphi, Y \cap E_j \ne \varphi$ and

2.14 $\quad (X \cap Y) \cap E_j \ne \varphi$ for j = 1,2,………., n

Prof: We have $(X \cap Y) \cap E_j \ne \varphi$ which gives $E_j \not\subset H(X \cap Y)$.

Also $X \cap E_j \ne \varphi$ and $Y \cap E_j \ne \varphi$ imply

$E_i \subset HX$ and $E_i \subset HY$, this yields $E_j \subset HX \cap HY$. Thus

$H(X \cap Y) \subsetneq HX \cap HY$

Conversely, suppose $H(X \cap Y) \subsetneq HX \cap HY$. Then there exists at least one E_j, $1 \le j \le n$, such that $E_j \subset HX \cap HY$ but $E_j \not\subset H(X \cap Y)$.

That is, $E_j \subseteq HX$ and $E_j \subseteq HY$ but $E_j \cap H(X \cap Y) = \varphi$

Then $E_j \cap X \ne \varphi$ and $E_j \cap Y \ne \varphi$ and $(X \cap Y) \cap E_j = \varphi$

This completes the proof.

Theorem 2.4: Let $E_1, E_2, \dots\dots E_n$ be a partition of an universe U. Let L, U be two unary dual operators on U satisfying (2.3) – (2.6) of definition 2.2. Then for any two subsets X,Y of U we have $H(X \cap Y) = HX \cap HY$ if and only if there exists on E_j for which conditions (2.13) and (2.14) hold.

Note. 4: Here also we get a theorem for upper approximation for the constructive approach of rough set.

Let $R : U \to U$ be an equivalence relation and let $E_1, E_2, \dots\dots E_n$ be a partition of U under the relation R. Then for two subsets $X, Y \subseteq U$, $\bar{R}(X \cap Y) = \bar{R}X \cap \bar{R}Y$,

if and only if there exists no E_j, $1 \le j \le n$, such that $X \cap E_j \ne \varphi$, $Y \cap E_j \ne \varphi$ and $(X \cap E) \cap E_j = \varphi)$ hold.

3. Rough Definability

We find four different types of rough sets defined as given below:

Let = (LX, HX) be a rough set on U. then

(a) Type-I: $\mathbb{X}$ is roughly LH –definable if $LX = \varphi$ and $HX \ne U$.

(b) Type-II: $\mathbb{X}$ is internally LH –undefinable if $LX = \varphi$ and $HX \ne U$.

(c) Type-III: $\mathbb{X}$ is externally LH –undefinable if $LX = \varphi$ and $HX = U$

(d) Type-IV: $\mathbb{X}$ is totally LH–undefinable if $LX = \varphi$ and $HX = U$.

If the $\mathbb{X}$ set is rough LH-definable, then we are able to decide for some element of U whether they belong to $\mathbb{X}$ or $\sim\mathbb{X}$. If is totally LH-undefinable, we are unable to decide for any element of U whether it belongs to $\mathbb{X}$ or $\sim\mathbb{X}$.

Other two classifications are in between type-I and type-IV. Moreover, external (or internal) LH-undefinable of a set $\mathbb{X}$ refers to the situation when positive (or negative) classification is possible for some objects, but it is impossible to decide that an object does not belong to $\mathbb{X}$ (or $\sim\mathbb{X}$).

We now find a theorem on LH-definability on the complement of:

Theorem 3.1: Let = (LX, HX) be a rough set on U. Then

(a) If $\mathbb{X}$ is roughly LH –definable then $\sim\mathbb{X}$ is roughly LH-definable.

(b) If $\mathbb{X}$ is internally LH –undefinable $\sim\mathbb{X}$ is externally LH-undefinable.

(c) If is externally LH –undefinable then $\sim\mathbb{X}$ is internally LH-undefinable

(d) If is totally LH –undefinable then $\sim\mathbb{X}$ is totally LH-undefinable.

Proof:

(a) As is roughly LH –definable then we have $LX = \varphi$ and $HX \neq U$. So that $L \sim X = HX \neq \varphi$ and $\sim LX \neq U$. Thus is roughly LH-definable. Hence the prove.

(b) Suppose is internally LH–undefinable then $LX = \varphi$ and $HX \neq U$. Hence $L \sim X = \sim HX = \varphi$ and $H \sim X = \sim LX = U$ then $\sim \mathbb{X}$ is externally LH-undefinable. This completes the prove.

In the similar manner we can prove (c) and (d). Hence we get the table of compl$_{\text{ement as}}$

Table. 1

$\mathbb{X}$	$\sim\mathbb{X}$
Type-I	Type-I
Type-II	Type-III
Type-III	Type-II
Type-IV	Type-IV

Theorem 2(a): If and are roughly LH-definable the $\mathbb{X} \cap \mathbb{Y}$ is either roughly LH-definable of internally LH-undefinable.

Proof: We have $LX \neq \varphi$, $HX \neq U$ and $LY \neq \varphi$, $HY \neq U$ as $\mathbb{X}$ and $\mathbb{Y}$ are roughly LH-definable. The

$L(X \cap Y) = LX \cap LY \neq \varphi$ or $= \varphi$ and

$H(X \cap Y) \subseteq HX \cap HY \neq U$. Hence $\mathbb{X} \cap \mathbb{Y}$ is roughly LH-definable or is internally LH-undefinable.

We note that if both and $\mathbb{Y}$ be the rough sets of type-I then $\mathbb{X} \cap \mathbb{Y}$ is a rough set of type-I or type-II.

Theorem 2(b): If $\mathbb{X}$ is roughly LH-definable and $\mathbb{Y}$ is totally LH-undefinable, then $\mathbb{X} \cap \mathbb{Y}$ is externally LH-undefinable.

Proof : We have $\mathbb{X}$ is roughly LH-Definable $LX \neq \varphi$ and $HX \neq U$ and $\mathbb{Y}$ is totally LH-undefinable $\Rightarrow LY = \varphi$, $HY = U$. So that $L(X \cap Y) = LX \cap LY = \varphi$ and $H(X \cap Y) \subseteq HX \cap HY \neq U$. Thus $\mathbb{X} \cap \mathbb{Y}$ is externally LH-undefinable and hence proved.

At that time we say $\mathbb{X} \cap \mathbb{Y}$ is a rough set of type-II whenever $\mathbb{X}$ is rough set of type-I and is a rough set of type-IV. Hence we find a table for intersection as

Table. 2

$\mathbb{X} \cap \mathbb{Y}$		$\mathbb{Y}$			
	$\cap$	Type-I	Type-II	Type-III	Type-IV
	Type-I	Type-I/Type-II	Type-II	Type-I/Type-II	Type-IV
	Type-II	Type-II	Type-II	Type-II	Type-II
	Type-III	Type-I/ Type-II	Type-I	Type-I/II/III/IV	Type-I/ Type-IV
$\mathbb{X}$	Type-IV	Type-II	Type-II	Type-II/Type-IV	Type-II/ Type-IV

Theorem 3.3

(a) If two set $\mathbb{X}$ and $\mathbb{Y}$ are roughly LH-definable then $\mathbb{X} \cup \mathbb{Y}$ is roughly LH-definable or is externally LH-undefinable.

(b) If is roughly LH-definable and $\mathbb{Y}$ is totally LH-undefinable then $\mathbb{X} \cup \mathbb{Y}$ is externally LH-undefinable.

Proof: (a) From the hypothesis , we define $LX \neq \varphi$, $HX \neq U$ and $LY \neq \varphi$, $HY \neq U$. Then $L(X \cap Y) \supseteq LX \cup LY \neq \varphi$ $H(X \cup Y) = HX \cup HY \neq U$ or U. Thus $\mathbb{X} \cup \mathbb{Y}$ is roughly LH-definable or externally LH-undefinable.

Hence $\mathbb{X} \cup \mathbb{Y}$ is a rough set of Type-I or Type-III whenever $\mathbb{X}$ and $\mathbb{Y}$ are both the rough sets of Type-I.

(b) From the hypothesis is roughly LH-definable $\Rightarrow LX \neq \varphi$ and $HX \neq U$ and $\mathbb{Y}$ is totally LH-undefinable $\Rightarrow LY = \varphi$ and $HY = U$. Thus $L(X \cup Y) \supseteq LX \cup LY \neq \varphi$. $H(X \cup Y) = HX \cup HY = U$ and hence the theorem.

Now $\mathbb{X}$ is rough set of Type-I and $\mathbb{Y}$ is of Type-IV, $\mathbb{X} \cup \mathbb{Y}$ is a roughest of Type-III.

We find the table for union as

Table. 3

$\mathbb{X} \cup \mathbb{Y}$		$\mathbb{Y}$			
	$\cup$	Type-I	Type-II	Type-III	Type-IV
$\mathbb{X}$	Type-I	Type-I/Type-III	Type-I/Type-III	Type-III	Type-IV
	Type-II	Type-I/Type-III	Type-I/Type-II/ Type-III/Type-IV	Type-III	Type-III/ Type-IV
	Type-III	Type-III	Type-III	Type-III	Type-III
	Type-IV	Type-III	Type-III/ Type-IV	Type-III	Type-III/ Type-IV

Theorem 3.4 : If $\mathbb{X}$ and $\mathbb{Y}$ are both the rough sets of Type-I then the difference $\mathbb{X} - \mathbb{Y}$ is a rough set of Type-I or Type-II.

Proof: From the given condition we find $LX \neq \varphi$ and $HX \neq U$ and $LY \neq \varphi$, $HY \neq U$. Then $L(X - Y) = L(X \cap \sim Y) = LX \cap (L(\sim Y) = LX \cap (\sim HY) = LX - HY = \varphi$ or $\neq \varphi$ and $H(X - Y) = H(X \cap \sim Y) \subseteq HX \cap H(\sim Y) = HX \cap (\sim LY) = HX - LY \neq U$.

Then $\mathbb{X} - \mathbb{Y}$ is a rough set of Type-I or Type-II. Hence the theorem. We get the table of difference as

Table. 4

$\mathbb{X} - \mathbb{Y}$		$\mathbb{Y}$			
	–	Type-I	Type-II	Type-III	Type-IV
$\mathbb{X}$	Type-I	Type-I/Type-II	Type-I/Type-II	Type-II	Type-II
	Type-II	Type-II	Type-II	Type-II	Type-II
	Type-III	Type-I/Type-II	Type-I/Type-II/ Type-III/Type-IV	Type-III Type-IV	Type-II/
	Type-IV	Type-II	Type-II/ Type-IV	Type-II	Type-II/ Type-IV

4. DE-MORGAN'LAW

The following tables yield the validity of De Morgan's Law.

Let $\mathbb{X} = (LX, HX)$ and $\mathbb{Y} = (LY, HY)$ be two rough sets on U. Then

(a) $\sim(\mathbb{X} \cap \mathbb{Y}) = \sim\mathbb{X} \cup \sim \mathbb{Y}$ (b) $\sim(\mathbb{X} \cup \mathbb{Y}) = \sim\mathbb{X} \cap \sim \mathbb{Y}$. To verify (a)

Table. 5

$\sim(\mathbb{X} \cap \mathbb{Y})$		$\mathbb{Y}$			
		Type-I	Type-II	Type-III	Type-IV
$\mathbb{X}$	Type-I	Type-I/ Type-III	Type-III	Type-I/ Type-III	Type-III
	Type-II	Type-III	Type-III	Type-III	Type-III
	Type-III	Type-I/ Type-III	Type-III	Type-I/Type-II Type-III/ Type-IV	Type-III/ Type-IV
	Type-IV	Type-III	Type-III	Type-III/ Type-IV	Type-III/ Type-IV

Table. 6

$\sim\mathbb{X} \cup \sim \mathbb{Y}$					
		Type-I	Type-II	Type-III	Type-IV
$\mathbb{X}$	Type-I	Type-I/Type-III	Type-III	Type-I/ Type-III	Type-III
	Type-II	Type-III	Type-III	Type-III	Type-III
	Type-III	Type-I/Type-III	Type-III	Type-I/ Type-II Type-III/ Type-IV	Type-III/ Type-IV
	Type-IV	Type-III	Type-III	Type-III/ Type-IV	Type-III/ Type-IV

5. CONCLUSION

The constructive approach to rough set , defined by Z.Pawlak,is based on a binary relation and is defined by lower and upper approximation operators. The constructive approach is more suitable for practical applications of rough sets, while the algebraic approach to rough set is appropriate for studying the structures of rough set algebra. The algebraic approach deals with axioms that must be satisfied by approximation operators without explicitly referring to a binary relation. The algebraic approach is more general and interesting than the constructive.

REFERENCES

[1] Iwinski, T : "Algebraic Approach to Rough Sets", Bult.Polish Academy Sc. And Math, 35, pp .673-738., 1987.

[2] Pawlak, Z : "Rough Sets" International Journals of Computer Sc. and Information Science II, pp.341-356, 1982.

[3] Pawalk, Z: "Rough Set, Theoritical Aspects of Reasoning about Data" 1991, Kluwer Academic Publishers.

[4] Tripathy, B.K., JOjha, J. and Mohanty, D. "On Rough Definability and Types of Approximation of Classifications", IEEE International Advance Computing Conference 6-7, March 2009, Patiala, India, 2009.

[5] Zadeh, L.A. "Fuzzy Sets", Information and Control, 8, pp.338-353, 1965.

[6] Yao,Y.Y, "Constructive and Algebraic Method of Theory of Rough Sets", Journal of Information Science, 109, pp.21-47, 1998.