

# TRAFFIC GROOMING, ROUTING AND WAVELENGTH ASSIGNMENT IN OPTICAL WDM MESH NETWORKS

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In this paper, we consider the traffic grooming routing, and wavelength assignment (GRWA) problem for optical mesh networks. In most previous studies on optical mesh networks, traffic demands are usually assumed to be wavelength demands, in which case no traffic grooming is needed. In practice, optical networks are typically required to carry a large number of lower rate (sub-wavelength) traffic demands. Hence, the issue of traffic grooming becomes very important since it can significantly impact the overall network cost. In our study, we consider traffic grooming in combination with traffic routing and wavelength assignment. Our objective is to minimize the total number of transponders required in the network. We first formulate the GRWA problem as an integer linear programming (ILP) problem. Unfortunately, for large networks it is computationally infeasible to solve the ILP problem. Therefore, we propose a decomposition method that divides the GRWA problem into two smaller problems: the traffic grooming and routing problem and the wavelength assignment problem, which can then be solved much more efficiently. In general, the decomposition method only produces an approximate solution for the GRWA problem.

However, we also provide some sufficient condition under which the decomposition method gives an optimal solution. Finally, some numerical results are provided to demonstrate the efficiency of our method.

Keywords: System Design, Mathematical Programming/Optimization

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## 1. INTRODUCTION

Wavelength division multiplexing (WDM) is now being widely used for expanding capacity in optical networks. In a WDM network, each fiber link can carry high-rate traffic at many different wavelengths, thus multiple channels can be created within a single fiber. There are two basic architectures used in WDM networks: ring and mesh. The majority of optical networks in operation today have been built based on the ring architecture. However, carriers have increasingly considered the mesh architecture as an alternative for building their next generation networks. Various studies have shown that mesh networks have a compelling cost advantage over ring networks. Mesh networks are more resilient to various network failures and also more flexible in accommodating changes in traffic demands. In order to capitalize on these advantages, effective design methodologies are required.

In the design of an optical mesh network, traffic grooming, routing, and wavelength assignment are some of the most important issues that need to be considered. The problem of traffic grooming and routing for mesh networks is to determine how to efficiently route traffic demands and at the same time to combine lower-rate (sub-wavelength) traffic demands onto a single wavelength. On the other hand, the problem of wavelength assignment is to

determine how to assign specific wavelengths to light paths, usually under the wavelength continuity constraint. In previous studies on the routing and wavelength assignment (RWA) problem (e.g., see [17, Chapter 8] and references therein), the issue of grooming has largely been ignored, i.e., it has been assumed that each traffic demand takes up an entire wavelength.

In practice, this is hardly the case, and networks are typically required to carry a large number of lower rate (sub wavelength) traffic demands. The traffic grooming problem has been considered by several researchers for ring networks (e.g. see, [1], [2], [3], and is only considered recently in [3] for mesh networks. The objective considered in [3] is either to maximize the network throughput or to minimize the connection-blocking probabilities, which are operational network-design problems. Alternatively, a strategic network-design problem is to minimize the total network cost.

Typically, the cost of a nation-wide optical network is dominated by optical transponders and optical amplifiers. If one assumes that the fiber routes are fixed, then the amplifier cost is constant, in which case one should concentrate on minimizing the number of transponders in the network. Grooming costs should also be considered. However, under realistic assumptions of either a low-cost interconnect between grooming equipment and transport equipment, or integrated (long-reach) transponders on the grooming equipment, then the relative cost of the grooming switch fabric is negligible, and minimizing the number of

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transponders is still the correct objective. In addition, the advent of Ultra Long-Haul transmission permits optical pass-through at junction nodes, hence, requiring transponders only at the end of the light paths.

Though the number of transponders has been used as an objective function in many studies on ring networks, it has not been considered at all for mesh networks. The objective functions that have been considered for mesh networks so far include: the blocking probability, the total number of wavelengths required, and the total route distance.

In this paper, we consider the problem of traffic grooming, routing, and wavelength assignment (GRWA) with the objective of minimizing the number of transponders in the network. We first formulate the GRWA problem as an integer linear programming (ILP) problem. Unfortunately, the resulting ILP problem is usually very hard to solve computationally, in particular for large networks. To overcome this difficulty, we then propose a decomposition method that divides the GRWA problem into two smaller problems: the traffic grooming and routing (GR) problem and the wavelength assignment (WA) problem. In the GR problem, we only consider how to groom and route traffic demands onto light paths (with the same objective of minimizing the number of transponders) and ignore the issue of how to assign specific wavelengths to light paths. Similar to the GRWA problem, we can formulate the GR problem as an ILP problem. The size of the GR ILP problem is much smaller than its corresponding GRWA ILP problem. Furthermore, we can significantly improve the computational efficiency for the GR ILP problem by relaxing some of its integer constraints, which usually leads to quite good approximate solutions for the GR problem. Once we solve the GR problem, we can then consider the WA problem, in which our goal is to derive a feasible wavelength assignment solution.

We note that the WA problem has been studied by several researchers before (e.g., see [1], [2], [3] and references therein). However, the objective in all these studies has been to minimize the number of wavelengths required in a network, in some cases by using wavelength converters. In general, the use of additional wavelengths in a network only marginally increases the overall network cost as long as the total number of wavelengths used in the network does not exceed a given threshold (the wavelength capacity of a WDM system). This is mainly because the amplification cost is independent of the number of wavelengths. In recent years, the wavelength capacity for optical networks has increased dramatically.

For example, with most advanced techniques, a single WDM system on a pair of fibers can carry up to 160 10G-wavelengths or 80 40G-wavelengths. Of course, once the wavelength capacity is exceeded, then a second parallel

system (with another set of optical amplifiers) needs to be built, which would then substantially increase the network cost.

Therefore, assuming a single WDM system on all fiber routes fixes the amplifier cost, and then one should focus on minimizing the number of transponders in the network, which is already taken into consideration in the GR problem. In this setting, the objective in our WA problem is to find a feasible wavelength assignment solution under the wavelength capacity constraint.

It is clear that in general the decomposition method would not yield the optimal solution for the GRWA problem. However, we will provide a sufficient condition under which we show that the decomposition method does produce an optimal solution for the GRWA problem. This is achieved by developing a simple algorithm that, under this sufficient condition, finds an optimal wavelength assignment.

The rest of this paper is organized as follows. In Section 2, we present the GRWA problem and demonstrate how it can be formulated as an ILP problem. In section 3, we first present our decomposition method. We then provide an ILP formulation for the GR problem and develop an algorithm for solving the WA problem. We also discuss under what condition the decomposition method produces an optimal solution for the GRWA problem. Some numerical results are provided in Section 4. Finally, a conclusion is given in Section 5.

## 2. THE GRWA PROBLEM

An optical network architecturally has two layers: a physical layer and an optical layer. The physical layer consists of fiber spans and nodes and the optical layer consists of light paths (optical links) and a subset of nodes contained in the physical layer. A light path in the optical layer is a path connecting a pair of nodes via a set of fiber spans in the physical layer.

Throughout this paper, we assume that light paths and their Routes in the physical layer are given. In practice, the selection of light paths is another important design issue that needs to be addressed, which is beyond the scope of this paper.

We use graph  $G_f = (V_f, E)$  to represent the physical layer, where  $E$  is the set of edges representing fiber spans and  $V_f$  is the set of nodes representing locations which are connected via fiber spans. We use graph  $G_o = (V_o, L)$  to represent the optical layer, where  $L$  is the set of edges representing light paths and  $V_o \subseteq V_f$  is a subset of locations that are connected via light paths. Each edge in  $L$  corresponds to a path in  $G_f$ . In this paper, we treat each light path as a logical connection between a pair of nodes (not just a single wavelength), therefore, one light path can contain multiple wavelengths. For ease of exposition, we

first assume that  $G_0$  is a directed graph (i.e., the light paths are unidirectional). The extension to the undirected graph case is quite straightforward and will be discussed later in this section (basically, we can simply replace every undirected edge with two directed edges).

The GRWA problem concerned in our study can be described as follows. Assuming that a set of traffic demands are given (some of them are of low rate, i.e., sub-wavelength), our goal is to find an optimal way to route and groom these demands in the optical layer,  $G_0$ , and also to assign a set of specific wavelengths to each lightpath so that the total number of transponders required is minimized.

There are two key constraints we need to take into consideration in this problem:

1. the wavelength capacity constraint for each fiber span, and
2. the wavelength continuity constraint for every lightpath, i.e., the same wavelength(s) needs to be assigned to a lightpath over the fiber spans it traverses. In this problem setting, the number of transponders required for each lightpath is equal to twice the number of wavelengths assigned to it (one transponder for each end of each wavelength on a lightpath). Therefore, by grooming several low rate demands onto a single wavelength, we can potentially reduce the total number of wavelengths required by the lightpaths, thus the number of transponders.

The GRWA problem can be formulated as an integer linear programming (ILP) problem. First, we need to introduce some necessary notation:

$W$ : the set of wavelengths available at each fiber;

$D$ : the set of traffic demands;

$s_d$ : the size of demand  $d \in D$ ;

$g$ : the capacity of a single wavelength;

$A$ :  $= [a_{v,l}] |V_0| \times |L|$ , the node-edge incidence matrix of graph;

$G_0$ , where  $a_{v,l} = 1$  if lightpath  $l$  originates from node  $v$ ,  $-1$  if lightpath  $l$  terminates at node  $v$ , and  $0$  otherwise;

$B$ :  $= [b_{e,l}] |E| \times |L|$ , the fiber-lightpath incidence matrix, where

$b_{e,l} = 1$  if fiber span  $e$  is on lightpath  $l$ , and  $0$  otherwise;

$u_d$ :  $= [u_{v,d}] |V_0|$ , the source-destination column vector for

$d \in D$ , where  $u_{v,d} = 1$  if  $v$  is the starting node of  $d$ ,  $-1$  if  $v$  is the end node of  $d$ , and  $0$  otherwise;

$x_d = [x_{l,d}] |L|$ , the column vector containing lightpath routing variables for  $d \in D$ , where  $x_{l,d} = 1$  if demand  $d$  traverses lightpath  $l$ , and  $0$  otherwise;  $y_w = [y_{l,w}] |L|$ , the column vector containing wavelength assignment variables for  $w \in W$ , where  $y_{l,w} = 1$  if wavelength  $w$  is assigned to lightpath  $l$ , and  $0$  otherwise (note that in our setting each lightpath  $l$  is treated as a logical connection between a pair of nodes, hence it can be assigned with multiple wavelengths, i.e., it is possible that  $\sum_w y_{l,w} \geq 1$ );

$\mathbf{1}$ :  $= [1, 1, \dots, 1]$ , the unit column vector.

Then the GRWA problem can be formulated as the following ILP problem (which we shall refer to as the GRWA ILP problem):

$$\min \sum_{w \in W, l \in L} y_{l,w}$$

$$s.t. \sum_{d \in D} A_{x,d} = u_d$$

$$By_w \leq \mathbf{1} \quad w \in W \quad (1)$$

$$\sum_{d \in D} s_d x_{l,d} \leq g \sum_{w \in W} y_{l,w} \quad l \in L \quad (2)$$

$$x_{l,d} \in \{0, 1\} \quad w \in W \quad (3)$$

$x$  and  $y$  are binary variables.

where the objective function  $\sum_{w \in W, l \in L} y_{l,w}$  is the total number of wavelengths assigned to all lightpaths, which is equivalent to minimizing the total number of transponders needed. The three constraints are:

- (1) is the flow balance equation, which guarantees that the lightpaths selected based on  $x_d$  constitute a path from the starting node of  $d$  to the end node of  $d$ .
- (2) implies a single wavelength along each fiber span can be assigned to no more than one lightpath.
- (3) is the capacity constraint for lightpath  $l$ , since  $\sum_{d \in D} s_d x_{l,d}$  is the total amount of demands carried by lightpath  $l$ , and  $g \sum_{w \in W} y_{l,w}$  is the total capacity of lightpath  $l$ .

We refer the type of the network considered above as the basic model. There are several variations of the basic model, which include:

1. Networks with both protected and unprotected demands;
2. Networks in which lightpaths are undirected;
3. Networks with non-homogeneous fibers where different types of fiber may have different wavelength capacities;
4. Networks in which demand exceeds a single WDM system per fiber pair.

### 3. A DECOMPOSITION METHOD

In the previous section, we formulated the GRWA problem as an ILP problem; however, it may not be computationally feasible to solve the ILP problem, particularly for large networks (e.g., see numerical results in Section 4). Therefore, it is necessary to find more efficient ways to solve the GRWA problem. In this section, we propose a decomposition method that divides the GRWA problem into two smaller problems: the traffic grooming and routing (GR) problem and the wavelength assignment (WA) problem. In the GR problem, we only consider how to groom and route demands over lightpaths and ignore the issue of how to assign specific wavelengths to lightpaths. Based on the grooming and routing, we can then derive wavelength capacity requirements for all lightpaths.

Similar to the GRWA problem, we formulate the GR problem as an ILP problem. The size of the GR ILP problem is much smaller than its corresponding GRWA ILP problem. Furthermore, we can significantly improve the computational efficiency for the GR ILP problem by relaxing some of its integer constraints, which usually leads to approximate solutions for the GR problem. Once we solve the GR problem, we can then consider the WA problem, in which our goal is to derive a feasible wavelength assignment solution that assigns specific wavelengths to lightpaths based on their capacity requirements derived in the GR problem.

It is obvious that in general the decomposition method would not yield the optimal solution for the GRWA problem.

However, we will provide a sufficient condition under which we show that the decomposition method does produce an optimal solution for the GRWA problem. We also develop a simple algorithm that finds a wavelength assignment solution under this sufficient condition.

#### A. The GR Problem

Let  $t = [t_l]_{l \in L}$ , a column vector containing lightpath capacity decision variables, where  $t_l = \sum_{w \in W} y_{lw}$ ,  $w$  is the number of wavelengths needed for lightpath  $l \in L$ . Then, the GR problem can be formulated as:

$$\begin{aligned} \min \quad & \sum_{l \in L} t_l \\ \text{s.t.} \quad & Ax_d = ud \quad d \in D \quad (4) \\ & Bt \leq |W|1 \quad (5) \\ & \sum_{d \in D} sdx_l, d \leq gt_l \quad l \in L \quad (6) \\ & d \in D \end{aligned}$$

$x$  binary variable and  $t$  integer variable.

We refer the above ILP problem as the GR ILP problem.

We now present the following result:

Proposition 1: If  $x$  and  $y$  are feasible solutions for the GRWA ILP problem, then  $x$  and  $t$  are feasible GRWA ILP problem,

$$\text{where } t = \sum_{w \in W} y_w$$

Proof: We first note that by summing over  $w \in W$  in (2) it leads to (5). Secondly, (3) is the same as (6). Hence, the result follows.

Based on Proposition 1, we have

Proposition 2: If  $x$  and  $t$  are the optimal solutions of the GR ILP problem, and there exists a binary  $y$  such that

$$\sum_{w \in W} y_w = t \text{ and } B y_w \leq 1 \text{ for } w \in W,$$

then  $x$  and  $y$  are the optimal solutions of the GRWA ILP problem.

Proof: Suppose  $x$  and  $y$  are feasible solutions for the GRWA ILP problem, then based on Proposition 1,  $x$  and  $t = \sum_{w \in W} y_w$  are feasible solutions for the GR ILP problem.

Since  $x$  and  $t$  are the optimal solutions of the GR ILP problem, we have

$$\sum_{w \in W, l \in L} y_{lw} = \sum_{l \in L} t_l \leq |L| \quad t_l = \sum_{w \in W, l \in L} y_{lw},$$

Therefore, the conclusion follows.

Obviously, the GR ILP problem is much easier to solve than the GRWA ILP problem since it has fewer integer variables and fewer constraints.

#### B. The WA Problem

The WA problem of our interest is to find a binary solution  $y$  such that  $\sum_{w \in W} y_w = t$  and  $B y_w \leq 1$

$$w \in W$$

for  $w \in W$ ,

where  $t$  is a feasible (or optimal) solution of the GR problem. This problem can be viewed as an ILP problem (without an objective function), which is much easier to solve than the GRWA ILP and the (relaxed) GR ILP problems.

Algorithm 1 (for the WA problem)

1. Select an initial lightpath  $l_0 \in L$  (arbitrarily), and assign a wavelength to  $l_0$ .

2. Suppose  $E_{i0} = \{e_1, e_2, \dots, e_k\}$ . Set  $L_0 = \{l_0\}$ . For  $i = 1$  to  $k$ , do

- a) Assign a wavelength to every lightpath  $l \in L_i \setminus L_j$   $0 \leq j < i$  such that no two lightpaths in  $L_i \setminus L_j$  share the same wavelength (note that  $L_i \setminus L_j$   $0 \leq j < i$

$L_j$  is a subset of lightpaths in  $L_i$  to which wavelengths have not been assigned yet).

- b) Let  $L_i = L_i \setminus \bigcup_{0 \leq j < i} L_j$ ,  
 $E_i = \bigcup_{l \in L_i} \{e_i\}$ .

We note that  $L_i$  is the set of lightpaths to which wavelengths are assigned in Step 2(a) and  $E_i$  is the set of fiber spans that are on at least one lightpath in  $L_i$  (excluding fiber span  $e_i$ ).

3. For  $i = 1, 2, \dots, k$ , apply the procedure in Step 2 to  $E_i$  (with  $E_{i0}$  being replaced with  $E_i$ ), and continue until all the lightpaths in  $L$  are assigned (note that since all the fiber spans in  $E_{i0}$  have been considered already in Step 2, we can simply replace  $E_i$  by  $E_i \setminus E_{i0}$ ).

#### 4. NUMERICAL RESULTS

In this section, we present one set of numerical example.

Example 1: This is relatively small network with 12 nodes, 17 fiber spans, 24 lightpaths, and 104 traffic demands (with different sizes). For this example, we were able to obtain the optimal solution based on the GRWA ILP formulation. The results are presented in Table 1

Table 1  
Numerical Results for Example 1

	Run Time	Solution
GRWA ILP	400 seconds	128
GR ILP 80	80 seconds	128
Relaxed GR ILP	2 seconds	136

#### 5. CONCLUSION

We studied the GRWA problem for optical mesh networks and proposed a decomposition method based on both ILP formulation and its relaxed version. In the decomposition method, we divided the GRWA problem into two smaller problems: the GR problem and the WA problem, both of which are much easier to solve compared to the original GRWA problem. We also provided a sufficient condition under which we proved that the decomposition method in fact produces an optimal solution for the GRWA problem. In general, our numerical results showed that the decomposition method produces quite good approximate solutions with relatively short run times and it can be used to solve the GRWA problem for large optical mesh networks (with a few hundred nodes and fiber spans).

#### REFERENCES

- [1] K. Zhu and B. Mukherjee, "Traffic Grooming in an Optical WDM Mesh Network," Proc. of IEEE International Conference on Communications, pp.721-725, Helsinki, Finland, June, 2001.
- [2] J.Q. Hu Sycamore Networks 15 St. Mary's Street Brookline, MA 02446 01824.
- [3] Brett Leida Boston University 220 Mill Road Chelmsford, MA.

