International Journal of Information Technology and Knowledge Management
July-December 2011, Volume 4, No. 2, pp. 379-383

CUBIC GRAPHS WITH EQUAL TWO DOMINATION NUMBER AND CHROMATIC NUMBER

G.Mahadevan[1], A.Selvam Avadyappan[2], A.Mydeen Bibi[3]

A Subset S of V is called a dominating set in G if every vertex in V-S is adjacent to at least one vertex in S. A Dominating set is said to be Two dominating set if every vertex in V-S is adjacent to atleast two vertices in S. The minimum cardinality taken over all, the minimal two dominating set is called two domination number and is denoted by γ_2 (G). The minimum number of colours required to colour all the vertices such that adjacent vertices do not receive the same colour is the chromatic number χ (G). In this paper, we investigate cubic graphs for which $\chi_2 = \chi = 3$.

Keywords: Two Domination Number, Chromatic Number. AMS Subject Classification: 05C

1. Introduction

Let G = (V, E) be a simple undirected graph. The degree of any vertex u in G is the number of edges incident with u and is denoted by d(u).The minimum and maximum degree of a vertex is denoted by δ(G) and Δ(G) respectively, P_n denotes the path on n vertices. The vertex connectivity κ(G) of a graph G is the Minimum number of vertices whose removal results in a disconnected graph. A colouring of a graph is an assignment of colours to its vertices so that two adjacent vertices have the same colour. An n-colouring of a graph G uses n colours. The Chromatic Number χ is defined to be the minimum n for which G has an n-colouring. If $\chi(G) = k$ but $\chi(G) < k$, for every proper subgraph H of G, then G is k-critical.

A subset S of V is called a dominating set in G if every vertex in V-S is adjacent to atleast one vertex in S. The minimum cardinality taken over all dominating sets in G is called the domination number of G and is denoted by γ. A dominating set is said to be Two Dominating set if every vertex in V-S is adjacent to atleast two vertices in S. The minimum cardinality taken over all, the minimal double dominating set is called Two Domination Number and is denoted by γ_2 (G).

Several authors have studied the problem of obtaining an upper bound for the sum of a domination parameter and a graph theoretic parameter and characterized the corresponding extremal graphs. In [11], Paulraj Joseph J and Arumugam S proved that $\gamma + k \leq p$. In [12] Paulraj Joseph J and Arumugam S proved that $\gamma_c + \chi = p + 1$. They also characterized the class of graphs for which the upper bound is attained. They also proved similar results for γ and γ_t. In[10], Paulraj Joseph J and Mahadevan G, proved that $\gamma_{cc} + \chi \leq 2n - 1$ and characterized the corresponding extremal graphs. In [13], Paulraj Joseph J and Mahadevan G proved that $\gamma_{pr} + \chi \leq 2n - 1$ and characterized the corresponding extremal graphs. In[7], Paulraj Joseph J and Mahadevan G proved that $\gamma_{ipr} + \chi \leq$"$2n - 2$ and characterized the corresponding extremal graphs. In [9], Paulraj Joseph J and Mahadevan G introduced the concept of complementary perfect domination number γ_{cp} and proved that $\gamma_{cp} + \chi \leq 2n - 2$, and characterized the corresponding extremal graphs. They also obtained the similar results for the induced complementary perfect domination number and chromatic number of a graph. In view of all the above, the authors have already obtained sharp upper bound for the sum of the two domination number and chromatic number and characterized the corresponding extremal graphs. Terms not defined here, are used in the sense of Harary[1]. There are several papers in which graphs with equal parameters and investigated. In [17] Volkman studied graphs for which $\gamma = \beta_1$. Paulraj Joseph J and Arumugam S investigated graphs for which $\gamma = \gamma_i$. Paulraj Joseph J analysed graphs for which the chromatic number equals domination parameters. In [6], Paulraj joseph and Mahadevan G investigated cubic graphs whose domination number equals chromatic number. Also J. Paulraj Joseph and G.Mahadevan, characterized the cubic graphs whose domination number equals the chromatic number equals to three. Motivated by the above, we now took the problem of characterizing the graphs for which two domination number equals to chromatic number equals to three.

In this paper, we investigate cubic graphs whose two domination number equals chromatic number equals three. We use the following results.

[1]Department of Mathematics, Anna University of Technology, Tirunelveli-627002, INDIA.

[2]Department of Mathematics V.H.N.S.N.College Virudhunagar, INDIA.

[3]Department of Mathematics, Sri Meenakshi Govt arts and Science College for women, Madurai-625002, INDIA.

Email: [1]gmaha2003@yahoo.co.in, [2]avadayappan@yahoo.co.in, [3]amydeen2006@yahoo.co.in

This document was created using SOLID CONVERTER PDF
To remove this message, purchase the product at www.SolidDocuments.com

Theorem 1.1 If G is a graph of order p, with maximum degree Δ, then $\gamma \leq [p/(\Delta + 1)]$.

Theorem 1.2 [Brook] If G is neither a complete graph nor an odd cycle, then $\chi \leq \Delta$.

Theorem 1.3 For any connected graph G, γ_2 (G) $\leq$ n.

Theorem 1.4 For any connected graph G, $\chi(G) \leq \Delta(G) + 1$

Let G = (V, E) be a connected cubic graph of order p with $\gamma_2 = \gamma$. By theorem 1.2, $\gamma \leq 3$. Clearly $\chi \neq 1$. We consider cubic graphs for which $\gamma_2 = \gamma = 3$. By theorem 1.1, $\gamma_2 \leq [p/4]$. Since $\gamma_2 = 3$, $6 \leq p \leq 15$ and $p \neq 14$. Since G is cubic, p is even. Hence p = 8, 10, 12.

2. CUBIC GRAPHS OF ORDER 8

Theorem 2.1 Let G be a connected cubic graph on 8 vertices. Then no graph exists for which $\gamma_2 = \chi = 3$.

Proof: Let S={u, v, w} be a minimum two dominating set of G and V – S ={x_1, x_2, x_3, x_4, x_5}. Clearly <S> ‘“ K_3. Hence we consider the three cases.

Case1: <S>= ‘$\bar{K}_3$

Without loss of generality, let u be adjacent to atleast one of the vertices of N(u) ={x_1, x_2, x_3}.

Subcase (a) Let v be adjacent to one vertex of N(u) say x_1. Then v is adjacent to x_4 and x_5. Now w is adjacent to x_1 or not adjacent to x_1. If w is adjacent to x_1, then w is adjacent to x_2 and x_3 (or equivalently x_4 and x_5), or x_2 (or equivalently x_3) and x_4 (or equivalently x_5).

If w is adjacent to x_2 and x_3, then x_2 is not adjacent to x_3. Hence x_2 must be adjacent to x_4(or equivalently x_5). Then {u, v, w} is not a two dominating set, which is a contradiction.

If w is adjacent to x_2 and x_4, then x_2 is not adjacent to x_4. Hence {u, v, w} is not a two dominating set, which is a contradiction.

If w is not adjacent to x_1 then without loss of generality, let w be adjacent to x_2, x_3 and x_4. Then {u, v, w} is not a two dominating set, which is a contradiction.

Subcase (b) Let v be adjacent to two vertices of N(u) say x_1 and x_2. Then without loss of generality, let v is adjacent to x_4. Now w is adjacent to x_1 (or equivalently x_2) or not adjacent to x_1 (or equivalently x_2)

If w is adjacent to x_1, then x_2 is adjacent to x_3 (or equivalently x_4) or w or x_5. If x_2 is adjacent to x_3, then w is not adjacent to x_3 and x_4. Also w is not adjacent to x_3 and x_5. Hence {u, v, w} is not a two dominating set, which is a contradiction.

If w is not adjacent to x_1 then without loss of generality, let w be adjacent to x_3, x_4 and x_5. Now x_5 is adjacent to x_3 and x_4 (or) x_1 and x_2 (or) x_1 and x_4 (or equivalently x_2 and x_3). In all the cases, {u, v, w} is not a two dominating set, which is a contradiction.

Subcase(c) Let v be adjacent to all the vertices of N(u), say x_1, x_2 and x_3. Now x_2 is adjacent to x_1(or equivalently x_3), or w (or equivalently x_4 or x_5). Since G is cubic, x_2 cannot be adjacent to x_1(or equivalently x_3). Hence x_2 must be adjacent to w.

If x_2 is adjacent to w then w is adjacent to x_1 and x_3 (or) x_4 and x_5 (or) x_1 and x_5. Since G is cubic, w cannot be adjacent to x_1 and x_3. Also w cannot be adjacent to x_1 and x_5. Hence w must be adjacent to x_4 and x_5. Then x_1 must be adjacent to x_4 and x_5 is adjacent to x_3 and x_4. Then {u, v, w} is not a two dominating set, which is a contradiction.

Case2: <S> = $K_2 \cup K_1$

Let uv be an edge. Without loss of generality, let u be adjacent to x_1 and x_2. Now w is adjacent to x_1, x_2 and anyone of {x_3, x_4, x_5}(or) w is adjacent to x_1(or equivalently x_2) and any two of {x_3, x_4, x_5}. If w is adjacent to x_1, x_2 and x_3, then x_4 is adjacent to x_1(or equivalently x_2) or not adjacent to x_1 (or equivalently x_2). If x_4 is not adjacent to x_1 (or equivalently x_2), then x_4 is adjacent to x_1, then x_5 is adjacent to x_4 or not adjacent to x_4. If x_5 is not adjacent to x_4, then x_5 is adjacent to x_2, x_3 and v. Thus in all the above cases, {u, v, w} is not a two dominating set which is a contradiction.

If w is adjacent to x_1,x_3 and x_4, then x_5 is adjacent to x_1 or not adjacent to x_1. If x_5 is adjacent to x_1, then {u, v, w} is not a two dominating set, which is a contradiction. If x_5 is not adjacent to x_1, then x_5 is adjacent to any three of {x_3, x_4, v, x_2}. Let x_5 be adjacent to x_3, x_4 and v. Then {u, v, w} is not a two dominating set, which is a contradiction.

Case 3: <S> = P_3

Let v be adjacent to u and w. Then without loss of generality, let v be adjacent to x_1. Now x_2 is adjacent to u, w and anyone of {x_3, x_4, x_5} or w and any two of {x_3, x_4, x_5}. If x_2 is adjacent to u, w and x_3 then x_4 is adjacent to w (or equivalently u) or not adjacent to w (or equivalently u). If x_4 is not adjacent to w, then x_4 is adjacent to x_5, x_3 and x_1. So that {u, v, w} is not a two dominating set, which is a contradiction. If x_4 is adjacent to w, then x_5 is adjacent to x_4 or not adjacent to x_4.

If x_5 is not adjacent to x_4, then x_5 is adjacent to x_1, u and x_3 , so that {u, v, w} is not a two dominating set, which is a contradiction. If x_5 is adjacent to x_4 then {u, v, w} is not a two dominating set, which is a contradiction . If x_2 is adjacent to w, x_3 and x_4 then {u, v, w} is not a two dominating set, which is a contradiction.

In all the above cases, Since G is cubic, no graph exists.

This document was created using SOLID CONVERTER PDF To remove this message, purchase the product at www.SolidDocuments.com

3. CUBIC GRAPHS OF ORDER 10

Theorem 3.1: Let G be connected cubic graph on 10 vertices. Then no graph exists for which $\gamma_2 = \chi = 3$.

Proof : Let S = {u, v, w} be a minimum two dominating set. Let $S_1 = N(u) = \{x_1, x_2, x_3\}$. Clearly $<S> \neq K_3$ or P_3. Hence $<S> = K_2 \cup K_1$ or ‘$\overline{K}_3$. If $<S> = K_2 \cup K_1$ then vw be the edge in <S>. Let x_6 and x_7 be the two remaining vertices adjacent to v and also x_4 and x_5 be the remaining two vertices adjacent to w. Let $S_2 = \{x_6, x_7\}$ and $S_3 = \{x_4, x_5\}$.

Lemma 3.1: If$<S> = K_2 \cup K_1$ and $<S_1> = P_3$, then no graph exists.

Proof: Without loss of generality, Let $<S_1> = P_3 = \{x_1, x_2, x_3\}$. We consider the following three cases.

Case 1: $<S_2> = <S_3> = K_2$.

Without loss of generality, let x_1 be adjacent to x_6. Since G is cubic, x_1 is not adjacent to atleast two vertices in S. Hence {u, v, w} is not a two dominating set, which is a contradiction.

Case : $<S_2> = K_2$ and $<S_3> = \overline{K}_2$

Let x_5 is adjacent to x_6 and x_7 or x_1 and x_3 or x_6 (or equivalently x_7) and x_4(or equivalently x_3). In all the above cases, since G is cubic, it does not satisfy the two dominating set, which is a contradiction.

Case : $<S_2> = <S_3> = \overline{K}_2$

Without loss of generality, Let x_1 be adjacent to x_6. Since G is cubic. Hence x_1 is not adjacent to atleast two vertices in S Hence {u, v, w}is not a two dominating set, which is a contradiction.

In all the above cases, Since G is cubic, no graph exists.

Lemma 3.2: If $<S>= K_2 \cup K_1$ and $<S_1> = \overline{K}_3$, then no graph exists.

Proof: Without loss of generality, Let$<S_1>=$ ‘$\overline{K}_3 = \{x_1, x_2, x_3\}$. We consider the following three cases.

Case 1: $<S_2> = <S_3> = K_2$

Without loss of generality, let x_1 be adjacent to x_6. Since G is cubic and $\{x_1, x_1, x_3\}$ is not adjacent to atleast two vertices in S. Hence {u, v, w} is not a two dominating set, which is a contradiction. Hence no graph exists in this case.

Case 2: $<S_2> = K_2$ and $<S_3> = \overline{K}_2$

Since G is cubic, every vertex in V-S is adjacent to atleast two vertices in S. This is impossible in this case. Hence {u, v, w} is not a two dominating set, which is a contradiction.

Case 3: $<S_2> = <S_3> = \overline{K}_2$

Since G is cubic as in the above case, {u, v, w} is not a two dominating set, which is a contradiction.

In all the above cases, Since G is cubic, no graph exists.

Lemma 3.3: If $<S> = K_2 \cup K_1$ and $<S_1> = K_2 \cup K_1$. Then no graph exists.

Proof: Let x_1x_2 be edge in $<S_1>$ we consider the following cases.

Case 1: $<S_2> = <S_3> = K_2$

Now x_3 is adjacent to x_5 and x_7 (or equivalently x_4 and x_5) or x_4 and x_6 (or equivalently x_5 and x_7). If x_3 is adjacent to x_6 and x_7, then x_2 is adjacent to x_4 (or equivalently x_5) and then x_1 is adjacent to x_5. Hence {u, v, w} is not a two dominating set, which is a contradiction.

Case 2: $<S_2> = K_2$ and $<S_3> = \overline{K}_2$

If x_3 is adjacent to x_4 and x_5 then x_4 is adjacent to x_1 (or equivalently x_2) or x_6 (or equivalently x_7). In all the case, {u, v, w} is not a two dominating set, which is a contradiction.

Case 3: $<S_2> = <S_3> = \overline{K}_2$

Now x_3 is adjacent to x_4 and x_5 (or equivalently x_6 and x_7) or x_4 and x_6(or equivalently x_5 and x_7). In all the cases {u, v, w} is not a two dominating set, which is a contradiction.

In all the above cases, Since G is cubic, no graph exists.

Lemma 3.4: If $<S> = \overline{K}_3$, v can be adjacent to two of the three vertices not in N(u). It is adjacent to two vertices say x_4 and x_5. Let $S_2 = \{x_4, x_5\}$, then w be adjacent to two other vertices say x_6, x_7. Also let $S_3 = \{x_6, x_7\}$.

If $<S> =$ ‘$\overline{K}_3$ and $<S_1> = P_3$, then no graphs exists.

Proof: Without loss of generality, let $<S_1> = P_3 = \{x_1, x_2, x_3\}$. We consider the following three cases.

Case1: $<S_2> = <S_3> = K_2$.

Let $X = S_2 \cup \{v\}$ and $Y = S_3 \cup \{w\}$, then $<X> = <Y> = C_3$. $<S>= K_2 \cup K_1$ which falls under lemma 3.1.

Case2: $<S_2> = K_2$ and $<S_3> = \overline{K}_2$.

Let x_4 and x_5 be the edge in S_2. Since G is cubic, v cannot be adjacent to x_1 or x_3. If v is adjacent to x_6 (or equivalently x_7) then x_7 is adjacent to x_1 and x_3 (or) x_4 and x_5 (or) x_4 (or equivalently x_5) and x_1(or equivalently x_3).

Subcase (a) If x_7 is adjacent to x_1 and x_3 then x_6 is adjacent to x_4 (or equivalently x_5) and then w is adjacent to x_5. Hence {u, v, w} is not a two dominating set , which is a contradiction.

Subcase (b) If x_7 is adjacent to x_1 and x_4 then x_5 is adjacent to x_6 or w. If x_5 is adjacent to x_6 then w is adjacent to x_6, which is a contradiction. Hence {u, v, w} is not two dominating set.

If x_5 is adjacent to w then x_3 is adjacent to x_6. Hence {u, v, w} is not a two dominating set, which is a contradiction.

This document was created using SOLID CONVERTER PDF To remove this message, purchase the product at www.SolidDocuments.com

Subcase (c) If x_7 is adjacent to x_5 and x_4 then x_6 is adjacent to x_1(or equivalently x_3).

In an above all the cases, {u, v, w} is not a two dominating set which is a contradiction.

Case 3: $<S_2> = <S_3> = \bar{K}_2$

Now v is adjacent to x_1 (or equivalently x_3) or x_6 (or equivalently x_7). In both the cases, {u, v, w} is not a two dominating set, which is a contradiction.

In all the above cases, Since G is cubic, no graph exists.

Lemma 3. 5: If $< S > = \bar{K}_3$ and $<S_1> = K_2 \cup K_1$, then no graph exists.

Proof: Let x_1x_2 be the edge in $<S_1>$ we consider the following cases.

Case 1: $<S_2> = <S_3> = K_2$

Now x_1 is adjacent to any one of the vertices $\{x_4, x_5, v\}$ (or equivalently any one of $\{x_6, x_7, w\}$) without loss of generality, let x_1 be adjacent to v. Hence {u, v, w} is not a two dominating set, which is a contradiction.

Case 2: $< S_2> = K_2$ and $<S_3> = \bar{K}_2$

Now x_1 is adjacent to any one of $\{v, x_4, x_5\}$ (or) w (or) x_6 (or equivalently x_7). In all the above cases,{u, v, w} is not a two dominating set, since G is cubic, which is a contradiction.

Case 3 : $<S_2> = <S_3> = \bar{K}_2$

Let x_1 be adjacent to v (or equivalently w) or x_4 (or equivalently x_5) or x_6 (or equivalently x_7). In all the above cases no graph exists. Hence {u, v, w} is not a two dominating set, which is a contradiction. In all the above cases, Since G is cubic, no graph exists.

Lemma 3.6: If $< S > = \bar{K}_3$ and $<S_1> = \bar{K}_2$, then no graph exists.

Proof: Let x_4x_5 be the edge in $<S_2>$ and x_6x_7 be the edge in $< S_3>$. We consider all the three cases.

Case 1 : $< S_2> = < S_3 > = K_2$

Now x_1 is adjacent to any two of $\{v, x_4, x_5\}$ (or equivalently any two of $\{w , x_6 , x_7\}$) or any of $\{v, x_4, x_5\}$ and anyone of $\{w, x_6, x_7\}$. In all the above cases,{u, v, w} is not a two dominating set ,which is a contradiction.

Case 2 : $<S_2> = K_2$ and $< S_3 > = \bar{K}_2$

Let w be adjacent to any one of $\{x_1, x_2, x_4\}$. Let w be adjacent to x_4. In this case also {u, v, w} is not a two dominating set ,which is a contradiction.

Case 3 : $<S_2> = <S_3> = \bar{K}_2$

Now v is adjacent to x_1 (or equivalently x_3) or x_6 (or equivalently x_7). In both the cases {u, v, w} is not a two dominating set which is a contradiction.

In all the above cases, since G is cubic, no graph exists.

4. CUBIC GRAPHS ON 12 VERTICES

Theorem 4.1: Let G = (V, E) be a connected cubic graph on 12 vertices. Then no graph exists for which $\gamma_2 = \chi = 3$.

Proof: We observe that if $\gamma_2=3$, then every γ_2 set S is an independent efficient two dominating set. Let S = {u, v, w} and $<S_1> = N(u) = \{x_1, x_2, x_3\}$. Let $<S_2> = N(v) = \{x_4, x_5, x_6\}$ and $<S_3> = N(w) =\{x_7, x_8, x_9\}$. Then we consider the following cases,

Case 1: $<S_1> = <S_2> = <S_3> = P_3$.

Case 2: $<S_1> = <S_2> = P_3$ and $<S_3> = K_2 \cup K_1$.

Case 3: $<S_1> = <S_2> = P_3$, and $<S_3> = \bar{K}_3$

Case 4: $<S_1> = P_3$ and $<S_2> = <S_3> = K_2 \cup K_1$.

Case 5: $<S_1> = P_3$ and $<S_2> = \bar{K}_3$, and $<S_3> = K_2 \cup K_1$.

Case 6: $<S_1> = P_3$ and $< S_2 > = < S_3 > = \bar{K}_3$.

Case 7: $<S_1> = < S_2 > = K_2 \cup K_1$, $< S_3 > = \bar{K}_3$.

Case 8: $< S_1> = < S_2 > = < S_3 > = K_2 \cup K_1$.

Case 9: $<S_1> = <S_2> = <S_3> = \bar{K}_3$.

Case 10: $<S_1> = K_2 \cup K_1$, $<S_2> = <S_3> = \bar{K}_3$.

In all the above cases, since G is cubic, after a tedious calculations it can be verified that, no graph exists.

REFERENCES

[1] Harary F. (1972), “Graph Theory”, Addison Wesley Reading Mass.

[2] Haynes, Teresa W.(2001), “Paired Domination in Graphs”, Congr. Numer 150.

[3] Haynes, Teresa W. (2000), “Induced-Paired Domination in Graphs”, Arts Combin, 57 (2000), 111-128.

[4] Mahadevan.G and Mydeenbibi.A (2007), “Double Domination Number and Chromatic Number of a Graph”, Referred Proceedings of the National Conference NCMCM Conducted by PSG College of Technology, Coimbatore, Narosha publications India.P.182-190.

[5] Mahadevan.G and Mydeen bibi.A (2009), “Characterization of Two domination number and Chromatic number of a graph”, -preprint.

[6] Mahadevan. G and J.Paulraj Joseph(2001), “Cubic Graphs with Equal Domination Number and Chromatic Number”. Proceedings of the National Conference on Graph theory and its applications, Narosa Publications, India.pp.129-139.

[7] Mahadevan.G. (2005), “On Domination theory and related concepts in graphs”, Ph.D.,thesis Manonmaniam Sundaranar University, Tirunelveli. India.

[8] Mahadevan G, Selvam A, Paulraj Joseph J (2005), “Extended results on Complementary Connected Domination Number and Chromatic Number of a Graph”, Proceedings of the National Conference SACOEFERENCE Conducted by Dr. Sivanthi Aditanar college of Engineering, Tiruchendur, India.P.438-442.

This document was created using SOLID CONVERTER PDF
To remove this message, purchase the product at www.SolidDocuments.com

[9] Paulraj Joseph J. and Mahadevan.G. (2006), "On Complementary Perfect Domination Number of a Graph", Acta Ciencia Indica, 31 M, No, 2, 847

[10] Paulraj Joseph J .and Mahadevan G.(2003), "Complementary Connected Domination Number and Chromatic Number of a Graph", Referred Proceedings of the Second National Conference on Mathematical and Computational Models, Edited by Arulmozhi and Natarajan, Allied Publications, India.

[11] Paulraj Joseph J.and Arumugam S(1992), "Domination and Connectivity in Graphs", International Journal of Management and Systems, 8 no 3:233-236.

[12] Paulraj Joseph J. and Arumugam S (1999), "Domination and Colouring in Graphs", International Journal of Management and Systems, 15, no.1, 37-44.

[13] Paulraj Joseph J. and Mahadevan G (2006), "Paired Domination and Chromatic Number of a Traph", Acta Cienia Indica, 31 M, No. 2, 847.

[14] Tamizh Chelvam T. and Jaya Prasad B.(2002), "Complementary Connected Domination Number", International Journal of Management and Systems, 18, No.22

[15] Teresa W .Haynes, Stephen T. Hedemiemi and Peter J .Slater (1998), "Fundamentals of Domination in Gaphs", Marcel Dekker, Newyork.

[16] L.Volkman(1994): "On Graphs with Equal Domination and Covering Number", Discrete .Math., 51, 211-214

[17] L.Volkman(1995), "On Graphs with Equal Domination and Edge Independence Number", Ars Combin. 41 .45-56.

This document was created using SOLID CONVERTER PDF To remove this message, purchase the product at www.SolidDocuments.com