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Estimation of Reflection and Transmission Coefficients of Water Waves by the Finite Element Method

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Abstract: A numerical method is used to analyze the reflection and transmission coefficients of water waves over an arbitrary varying topography.Formulation of finite element method is made using the diffraction of waves in the finite depth of water. The model is applied to analyze waves due to an abrupt depth change. Obtained results are further compared with anaylical results and boundary element results.

Keywords: Scattering, finite element method, relection coefficient, transmission coefficient, water waves.

1 Introduction

The scattering of water waves over an submerged object has been frequently and widely studied through theoretical and numerical tools for its practical applications in coastal engineering. For example, in the process of oil and gas extraction near sea shore, the bottom of sea is not flat, it has arbitrary topography. Waves reached at the nearshore zone may cause the erosion and deposition as well as influence the safety of coastal structures.

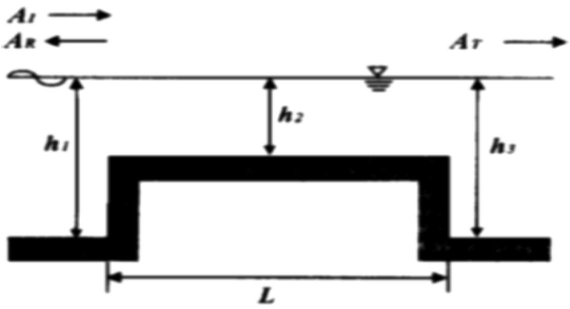
Among theoretical studies of surface waves, Dean [1] and then Ursell [2] calculated coefficients . For instance, Ursell [2] employed the singular integral equation approach in conjunction with Havelock's expansion to obtain an analytical solution of the problem in deep water. Miles [3] used a scattering matrix method to calculate the reflection and transmission coefficients for the case of a step discontinuity between two finite depths.

Liu and Abbaspour [4] studied the diffraction of obliquely incident waves by an infinite number of cylinders with the boundary integral equation method.O' Hare and Davies[5] presented a new method for the modeling of propagatin of waves over a smoothly varying bottom topography.

The model is developed using Galerkin finite element scheme over a shelf topographies and approximated results was compared to available theoretical results.

2 Mathematical formulation

Let the surface water waves is incident on the shelf topography. The problem is analyzed in two dimensional co-ordinate system (x,z). Let x-axis represents the undisturbed free surface and z-axis is vertically donward.



Under the assumption of incompressible and irrotational flow, and inviscid fluid, the fluid motion may be represented by a potential flow and the flow is governed by the Laplace equation as

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial z^2} = 0, 0 \le z \le h(x), \tag{1}$$

with boundary conditions

$$\frac{\partial\phi}{\partial z} + \frac{\sigma^2}{g}\phi = 0, \quad \text{on } z = 0 \tag{2}$$

$$\nabla\phi \cdot \hat{n} = 0$$

To estimate reflection and transmission coefficients of waves over an irregular bottom topograph, the far field condition is used

This relation can be written as

$$\phi(x, y) \to \begin{cases} \phi^{inc}(x, y) + R\phi^{inc}(-x, y) & \text{as} \quad x \to -\infty, \\ T\phi^{inc}(x, y) & \text{as} \quad x \to \infty, \end{cases}$$

along with these conditions at an abrupt depth physically the continuity of pressure and conservation of mass are employed.

3 Finite Element Formulation

We employ the Galerkin method among the weighted residue methods which force to be equal zero an error or residue occuring at each node as the approximate solution is substituted into the governing differential equation

$\phi = \sum_{i=1}^{n} \phi_i N_i$

where ϕ_i are nodel values and N_i are shape functions.

$$\begin{aligned} \text{Residue } R &= -\int_\Omega w\left[\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial z^2}\right] d\Omega \\ &= \int_\Omega \left[\frac{\partial w}{\partial x}\frac{\partial\phi}{\partial x} + \frac{\partial w}{\partial z}\frac{\partial\phi}{\partial z} - \left\{\frac{\partial}{\partial x}\left(w\frac{\partial\phi}{\partial x}\right) + \frac{\partial}{\partial z}\left(w\frac{\partial\phi}{\partial z}\right)\right\}\right] d\Omega \\ &= I_1 - I_2 \end{aligned}$$

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where

$$I_1 = \int_\Omega \left[\frac{\partial w}{\partial x}\frac{\partial \phi}{\partial x} + \frac{\partial w}{\partial z}\frac{\partial \phi}{\partial z} \right] d\Omega$$

$$I_2 = \left\{ \frac{\partial}{\partial x}\left(w\frac{\partial \phi}{\partial x} \right) + \frac{\partial}{\partial z}\left(w\frac{\partial \phi}{\partial z} \right) \right\}$$

Using Galerkin approach, we have $w = N_j \quad j = 1, 2...n$

$$I_1 = \sum_{i=1}^{n} \phi_i \int_\Omega \left[\frac{\partial N_j}{\partial x}\frac{\partial N_i}{\partial x} + \frac{\partial N_j}{\partial z}\frac{\partial N_i}{\partial z} \right] d\Omega$$

and

$$I_2 = \int_\Omega \left[\frac{\partial}{\partial x}\left(w\frac{\partial \phi}{\partial x} \right) + \frac{\partial}{\partial z}\left(w\frac{\partial \phi}{\partial z} \right) \right] d\Omega$$

$$= \int_\Gamma \left(w\frac{\partial \phi}{\partial x}n_x + w\frac{\partial \phi}{\partial z}n_z \right) d\Gamma \qquad \text{Using divergence theorem}$$

$$= \int_\Gamma w \left\{ -\frac{\partial \phi_{inc}}{\partial x} - ik_0\phi_{inc} \right\} d\Gamma + \int_\Gamma w \left\{ -K\phi \right\} d\Gamma$$

$$= \int_\Gamma N_j \left\{ -2\frac{\partial \phi_{inc}}{\partial x} \right\} d\Gamma - K\sum_{i=1}^{n} \phi_i \int_\Gamma N_j N_i \, d\Gamma$$

$$R = \sum_{i=1}^{n} \phi_i \left[\int_\Omega \left\{ \frac{\partial N_j}{\partial x}\frac{\partial N_i}{\partial x} + \frac{\partial N_j}{\partial z}\frac{\partial N_i}{\partial z} \right\} d\Omega + \int_\Gamma K N_j N_i \, d\Gamma \right] + \int_\Gamma N_j \left\{ 2\frac{\partial \phi_{inc}}{\partial x} \right\} d\Gamma$$

$$[K]\,[\phi] = [F]$$

we

4 Numerical results

In this section, for fixed values of parameter $h_1 = h_3$ and $h_2 = h_1/3, L = 10h_1$ the velocity potential $\phi(x, y)$ reflection coefficient $|R|$ and transmission coefficient $|T|$ are obtained for different values of wave number K.

Kh_1	k	$\phi(x, y)$	$\lvert R\rvert$
0.2	0.046267	-0.41467-2.6467 i	0.21347
0.45	0.072547	0.94823+2.50689 i	0.32111
0.75	0.0990179	0.20105+2.7365 i	0.27137
1.5	0.162181	-2.3011-1.7793 i	0.11218
2.75	0.277161	0.66574+2.6439 i	0.01251

Table 1: Numerical values of $\phi(x, y)$ and $|R|$ for different Kh_1 and $h_1 = 10$.

Kh_1	k	$\phi(x, y)$	$\lvert T\rvert$
0.1	0.160797	0.81243+0.12351 i	0.909162
0.25	0.260906	-0.67354-1.23457 i	0.93144
0.45	0.362735	0.42357+1.57676 i	0.97173
0.7	0.473805	2.00137+0.00412 i	0.94158
1.0	0.599839	1.99785+2.57100 i	0.91357

Table 2: Numerical values of $\phi(x, y)$ and $|T|$ for different Kh_1 and for $h_1 = 2$.

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5 Conclusion

The reflection and transmission coefficients of waves propagating over the shelf toprographies were calculated by using the Galerkin finite element method. It is also observed that the evanescent modes may play a significant role at the point where a water depth abruptly changes. Obtained results resonably agreed with eigen function expansion method for shelf topography.

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